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# Boundary state description of tachyon condensation

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Abstract: We construct the explicit boundary state description of the vortex-type (codimension two) tachyon condensation in brane-antibrane systems generalizing the known result of the kink-type (Frau et al. hep-th/9903123). In this description we show how the RR-charge of the lower-dimensional D-branes emerges. We also investigate the tachyon condensation in $\mathbb{T}^4/\mathbb{Z}_2$ orbifold and find that the twisted sector can be treated almost in the same way as the untwisted sector from the viewpoint of the boundary state. Further we discuss the higher codimension cases.

Keywords: Superstrings and Heterotic Strings, D-branes, Conformal Field Models in String Theory.

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## Contents



## 1. Introduction

Recently there have been tremendous progresses in understanding non-BPS configurations of D-branes and tachyon condensations in them, pioneered by Sen (for a review see [1]). In superstring theory most of these systems are realized as either brane-antibrane systems [2]–[5] or non-BPS D-branes [5]–[13]. The open strings between a Dp-brane and an anti-Dp-brane are projected by the GSO projection opposite to the usual cases and the tachyon survives. A non-BPS D-brane is defined as a D-brane without any GSO projections and the tachyonic instability also occurs. Sen argued that if the condensation of the constant tachyon field stabilizes the system, then the system will finally go down to the vacuum [4] and if the condensation has non-trivial configurations such as kinks or vortexes, then the final object will be D-branes of corresponding codimension [3, 5, 8, 14]. For example, the kink configuration in D$p$-$\overline{\mathrm{D}p}$ brane system is identified as a non-BPS D$(p-1)$-brane.

Three different approaches have been considered[1] to analyze those systems. The first one is to use conformal field theory descriptions of string world sheet with a boundary [5, 8, 9, 15, 14]. In this method, the tachyon condensation can be regarded as a marginal deformation of the boundary conformal field theory (BCFT) at a special radius. The second is the K-theory approach by Witten. He argued that the topological charges of lower D-branes in a non-BPS configuration of D-branes are classified by the corresponding K-group [17, 12]. In other words we can say that the topological configurations of tachyon fields one-to-one correspond to the element of the K-group. The third one is the string field theory description [18]. The tachyon potential has been calculated in the case of a D-brane in bosonic string [19] and a non-BPS D-brane in superstring [20]. The numerical results are in good agreement with the Sen's conjecture that the system at the minimum of the tachyon potential can be identified as the vacuum. The generations of lower-dimensional D-brane charges have been also discussed in this formalism [21].

In this paper we are interested in the first approach. From the viewpoint of the open strings the BCFT descriptions of tachyon kink condensations have been given in [5, 8, 9] for a brane-antibrane system or a non-BPS D-brane in the presence of the orbifold and orientation projection. In order to discuss the generation of codimension two D-branes, the vortex line configuration of the tachyon field is needed and is realized in [15] as a pair of the vortex and anti-vortex.

On the other hand we can use the boundary state formalism (for example see [22]), which can give more systematic CFT description of D-branes. In this formalism D-branes are constructed in the closed string Hilbert space. Therefore the couplings of D-branes to NSNS, RR-fields can be written down explicitly. The equivalence between the open string description and the closed string description should be required as in the usual BPS D-branes and this crucial constraint is called Cardy's condition [23]. The boundary state description of the tachyon kink pair in flat space was first constructed in [24]. Also in the case of the bosonic string the boundary state description was discussed in [25]. In the first half of this paper we extend this construction [24] to the case of tachyon vortex pairs in flat space. We construct explicitly the boundary state which describes the tachyon vortex pair condensation in D2-$\overline{\mathrm{D}2}$ system at the critical radius. The Cardy's condition is established and

[1] Quite recently a new approach which utilizes the non-commutative field theory description of the world volume theory has been considered in the presence of a large B-field [16].

the emergence of lower D-brane RR-charges is shown explicitly. At the point where the tachyon condensation is maximum the boundary state of the system is identified as that of a D0-$\overline{\text{D0}}$ system. Many other points correspond to the bound state of D2-$\overline{\text{D2}}$ and D0-$\overline{\text{D0}}$. Also the requirement[2] of the non-trivial "Chan-Paton factor of closed string" is verified in this formalism. Further we generalize these results into the higher codimension cases.

In the latter half we treat the case of $\mathbb{T}^4/\mathbb{Z}_2$ orbifold. Remarkably it is shown that the twisted sector boundary state can be written as the untwisted sector boundary state of another fields on the world sheet at the critical radius by performing bosonizations and fermionizations. Using this fact we can describe the tachyon kink condense explicitly starting from a D0-$\overline{\text{D0}}$ system and show that the untwisted RR charge vanishes and the twisted RR charge remains after the condensation, verifying the known identification [8] of the final object as a non-BPS D1-brane between the fixed points. This boundary state approach enables us to generalize the tachyon kink in the orbifold theory into the higher codimention cases such as the decay mode from D4-$\overline{\text{D4}}$ to D0-$\overline{\text{D0}}$, which has not been discussed before. We also discuss the relation between the bose-fermi degeneracy [26, 27] and the "bosonization procedure" in the boundary state description.

The paper is organized as follows. In section 2 we review the known results about the BCFT description of a tachyon vortex pair in a D2-$\overline{\text{D2}}$ system and a tachyon kink pair in a orbifolded D0-$\overline{\text{D0}}$ system. Further we investigate some details. In section 3 we construct the explicit boundary state which describes the condensation of the tachyon vortex pair in a D2-$\overline{\text{D2}}$ system. We show the final object can be identified with D0-$\overline{\text{D0}}$. Next we generalize the results and see that the tachyon condensation generates D-brane charges of higher codimension. In section 4 we study the tachyon kink in $\mathbb{T}^4/\mathbb{Z}_2$ orbifold. We give the corresponding boundary state and identify the final object. We also generalize the result into higher codimension cases. In section 5 we summarize the results and draw conclusions. In appendix A we give a brief review of boundary state and show our conventions. In appendix B we prove that the "bosonized" boundary state indeed satisfies the original boundary condition including detailed cocycle factors.

## 2. CFT description of tachyon condensation

In this section we review the descriptions of tachyon condensation from the viewpoint of open strings. These results are needed when we compare the results with those gained in the boundary state formalism. First we see the vortex-type tachyon condensation in the D2-$\overline{\text{D2}}$ system following [15, 5] and investigate some details in a slightly generalized situation. Next we review several known facts about the brane-

[2] In the case of a non-BPS D-brane the similar requirement was mentioned in [7, 1].

antibrane system in $\mathbb{T}^4/\mathbb{Z}_2$ orbifold and the tachyon condensation in that system. Such a system was first discussed in [8, 11] and also considered in [9] using the T-dualized picture. In this paper we consider type-II string theory only in the weak coupling region.

### 2.1 Tachyon condensation in a D2-$\overline{\mathrm{D2}}$ system

We take a parallel D2-brane and an anti D2-brane in type-IIA string theory along $x^1, x^2$ and compactify these directions on a torus of radii[3] $R_1 = 1$, $R_2 = 1$. Then we set a $\mathbb{Z}_2$ Wilson line along each circle. There are four types of Chan-Paton factors for the open strings in D2-$\overline{\mathrm{D2}}$ system and are denoted by $1, \sigma_1, \sigma_2, \sigma_3$ using Pauli matrices. We use $1, \sigma_3$ in order to represent the open strings with both ends on the same brane and the spectrum is determined by the conventional GSO projection. On the other hand $\sigma_1, \sigma_2$ correspond to the open strings with two ends on two different branes and follow the opposite GSO projection allowing the tachyon in the spectrum.

We consider the condensation of the following two types[4] of the tachyon field

$$T_{(1)}\left(x^1, x^2\right) = e^{i\frac{1}{2}(x^1+x^2)} - e^{-i\frac{1}{2}(x^1+x^2)}, \tag{2.1}$$

$$T_{(2)}\left(x^1, x^2\right) = -i\, e^{i\frac{1}{2}(x^1-x^2)} + i\, e^{-i\frac{1}{2}(x^1-x^2)}. \tag{2.2}$$

If we switch on only one of these, we get the tachyon kink configuration and a codimension one D-brane or a non-BPS D1-brane will be generated. On the other hand if we condense both at the same time, the tachyon vortex line pair configuration will lead to a pair of codimension two D-branes or a D0-$\overline{\mathrm{D0}}$ system.

The corresponding open string vertex operators in (0)-picture are written as

$$\begin{aligned} V_{T1} &= \left(\chi^1 + \chi^2\right)\left(e^{i\frac{1}{2}(X^1+X^2)} + e^{-i\frac{1}{2}(X^1+X^2)}\right) \otimes \sigma_1\,, \\ V_{T2} &= \left(\chi^2 - \chi^1\right)\left(e^{i\frac{1}{2}(X^1-X^2)} + e^{-i\frac{1}{2}(X^1-X^2)}\right) \otimes \sigma_2\,, \end{aligned} \tag{2.3}$$

where $X^i = X^i_R + X^i_L$, $\chi^i = \chi^i_R + \chi^i_L$ $(i = 1, 2)$ denote the bosonic fields on the string world sheet in NS-R formalism and their superpartners.

Notice that at the radii $R_1 = 1$, $R_2 = 1$ (critical radii) the lightest tachyon vertex operators become marginal owing to the Wilson lines and the tachyon condensation corresponding to such operators can be treated as the marginal deformation of CFT.

Now let us rotate the coordinates by $\pi/4$

$$\begin{aligned} Y^1 &= \frac{1}{\sqrt{2}}\left(X^1 + X^2\right), \quad Y^2 = \frac{1}{\sqrt{2}}\left(X^1 - X^2\right), \\ \psi^1 &= \frac{1}{\sqrt{2}}\left(\chi^1 + \chi^2\right), \quad \psi^2 = \frac{1}{\sqrt{2}}\left(\chi^1 - \chi^2\right). \end{aligned} \tag{2.4}$$

[3] In this paper we use $\alpha' = 1$ unit.

[4] There are also other two marginal deformations which represent other tachyon condensations. But these correspond to the shift of the vortex line center and the physical phenomena which occurs by such tachyon condensations do not change if we ignore these. Thus we only consider the tachyon fields (2.1) and (2.2) below.

This procedure enables us to use the method of bosonization and fermionization as follows:

$$\begin{aligned}
e^{i\sqrt{2}\,Y_R^i} &= \frac{1}{\sqrt{2}}\left(\xi_R^i + i\,\eta_R^i\right)\otimes\tau_i\,, & e^{i\sqrt{2}\,Y_L^i} &= \frac{1}{\sqrt{2}}\left(\xi_L^i + i\,\eta_L^i\right)\otimes\tau_i\,,\\
e^{i\sqrt{2}\,\phi_R^i} &= \frac{1}{\sqrt{2}}\left(\xi_R^i + i\,\psi_R^i\right)\otimes\tilde{\tau}_i\,, & e^{i\sqrt{2}\,\phi_L^i} &= \frac{1}{\sqrt{2}}\left(\xi_L^i + i\,\psi_L^i\right)\otimes\tilde{\tau}_i\,,\\
e^{i\sqrt{2}\,\phi_R'^i} &= \frac{1}{\sqrt{2}}\left(\eta_R^i + i\,\psi_R^i\right)\otimes\hat{\tau}_i\,, & e^{i\sqrt{2}\phi_L'^i} &= \frac{1}{\sqrt{2}}(\eta_L^i + i\psi_L^i)\otimes\hat{\tau}_i\,,
\end{aligned} \tag{2.5}$$

where $\tau_i$, $\tilde{\tau}_i$, $\hat{\tau}_i$ $(i=1,2)$ are cocycle factors [15, 8] and we also assume $\psi_{L,R}^i$, $\eta_{L,R}^i$ have the cocycle factor $\tau_3, \tilde{\tau}_3$. To be exact, other kinds of cocycle factors are needed in front of the exponential fields. The latter type of cocycle factors, which we will call second-type cocycle factors below, cannot be ignored when we later discuss the bosonizations and fermionizations of boundary states. We leave the details in the appendix B.

The operator product expansions (OPE) among these fields are[5]

$$\begin{aligned}
Y_R^i(z)Y_R^j(0) &\sim -\frac{1}{2}\,\delta_{ij}\ln z\,, & Y_L^i(\bar{z})Y_L^j(0) &\sim -\frac{1}{2}\,\delta_{ij}\ln\bar{z}\,,\\
\xi_R^i(z)\,\xi_R^j(0) &\sim \delta_{ij}\,\frac{i}{z}\,, & \xi_L^i(\bar{z})\,\xi_L^j(0) &\sim -\delta_{ij}\,\frac{i}{\bar{z}}\,,\\
\eta_R^i(z)\,\eta_R^j(0) &\sim \delta_{ij}\,\frac{i}{z}\,, & \eta_L^i(\bar{z})\,\eta_L^j(0) &\sim -\delta_{ij}\,\frac{i}{\bar{z}}\,.
\end{aligned} \tag{2.6}$$

Also the following identities are useful:

$$\begin{aligned}
\eta_R^i\,\xi_R^i &= i\sqrt{2}\,\partial Y_R^i\,, & \eta_L^i\,\xi_L^i &= -i\sqrt{2}\,\bar{\partial}Y_L^i\,,\\
\psi_R^i\,\xi_R^i &= i\sqrt{2}\,\partial\phi_R^i\,, & \psi_L^i\,\xi_L^i &= -i\sqrt{2}\,\bar{\partial}\phi_L^i\,.
\end{aligned} \tag{2.7}$$

Now we can express the tachyon vertex operators (2.3) in the following convenient way

$$\begin{aligned}
V_{T1} &= 2\,i\,\psi^1\xi^1\otimes\tau_2\otimes\sigma_1 = -2\sqrt{2}\,\partial\phi^1\otimes\tau_2\otimes\sigma_1\,,\\
V_{T2} &= -2\,i\,\psi^2\xi^2\otimes\tau_1\otimes\sigma_2 = +2\sqrt{2}\,\partial\phi^2\otimes\tau_1\otimes\sigma_2\,,
\end{aligned} \tag{2.8}$$

where $\partial$ denotes tangential derivative along the boundary. Then the tachyon condensation is represented as the insertion of the following Wilson lines in terms of the field $\phi$

$$\exp\left(\frac{i\,\alpha}{2\sqrt{2}}\oint\partial\phi^1\otimes\tau_2\otimes\sigma_1 + \frac{i\,\beta}{2\sqrt{2}}\oint\partial\phi^2\otimes\tau_1\otimes\sigma_2\right), \tag{2.9}$$

where $\oint$ denotes integration along the boundary and $\alpha, \beta$ mean parameters of tachyon condensations. Notice that $\tau_2\otimes\sigma_1$ commutes with $\tau_1\otimes\sigma_2$ and the above Wilson

[5] Note that the factors $i$ in the bosonic field OPE's are due to second-type cocycle factors.

line is well defined without path ordering. The open string spectrum in the R-sector does not change in the presence of the Wilson line because for the R-sector $\phi$ satisfies Neumann boundary condition at one end and Dirichlet boundary condition at the other end and there is no zero mode for $\phi$ [5]. Therefore we will investigate only the NS-sector.

Now let us define several projection operators in the following way:

$$\begin{aligned}
(-1)^F: \quad & |0\rangle \longrightarrow -|0\rangle\,, & \psi^i \longrightarrow -\psi^i\,, \qquad & (\xi^i,\eta^i) \longrightarrow (\xi^i,\eta^i)\,,\\
h_1: (\xi^1,\eta^1) & \longrightarrow -(\xi^1,\eta^1)\,, & (\xi^2,\eta^2) \longrightarrow (\xi^2,\eta^2)\,, \qquad & \psi^i \longrightarrow \psi^i\,,\\
Y^1_{L,R} & \longrightarrow Y^1_{L,R} + \frac{\pi}{\sqrt{2}}\,, & Y^2_{L,R} \longrightarrow Y^2_{L,R}\,, \qquad & \\
h_2: (\xi^1,\eta^1) & \longrightarrow (\xi^1,\eta^1)\,, & (\xi^2,\eta^2) \longrightarrow -(\xi^2,\eta^2)\,, \qquad & \psi^i \longrightarrow \psi^i,\\
Y^1_{L,R} & \longrightarrow Y^1_{L,R}\,, & Y^2_{L,R} \longrightarrow Y^2_{L,R} + \frac{\pi}{\sqrt{2}}\,. \qquad &
\end{aligned} \tag{2.10}$$

As is clear from the above definition, $(-1)^F$ is the fermion number on the world sheet and $h_1, h_2$ are the translation operators in the direction of $Y^1, Y^2$. We also define $(-1)^{F^\phi}, h_1^\phi, h_2^\phi$ similarly for $\phi$.

Since so far we have implicitly assumed the radius of circle in the direction of $Y^1$, $Y^2$ is $\sqrt{2}$, we should have a certain constraint in order to realize the physical periodicity $X^1 \sim X^1 + 2\,\pi$, $X^2 \sim X^2 + 2\,\pi$ taking the effect of the Wilson line into consideration. Such a constraint is given as

$$(-1)^F h_1 h_2 = (-1)^{F^\phi} h_1^\phi h_2^\phi = 1\,, \tag{2.11}$$

where we used eq. (2.10). There are eight sectors in NS-sector which survive this projection as follows:

$$\begin{aligned}
&1\otimes 1\,, \qquad 1\otimes\tau_3\,, \qquad \sigma_3\otimes 1\,, \qquad \sigma_3\otimes\tau_3\,,\\
&\sigma_1\otimes\tau_1\,, \qquad \sigma_1\otimes\tau_2\,, \qquad \sigma_2\otimes\tau_1\,, \qquad \sigma_2\otimes\tau_2\,.
\end{aligned} \tag{2.12}$$

Four of these are insensitive to the tachyon condense or equally the insertion of the Wilson lines. But the momenta of $\phi$ in the other four sectors are shifted in proportion to the deformation parameters $\alpha$, $\beta$. The details are shown in table 1. Note that $\alpha, \beta$ have periodicity $\alpha \sim \alpha + 2$, $\beta \sim \beta + 2$ by applying the same argument discussed in [5].

The main claim in [15] is that if the tachyon condensation develops into the point $\alpha = 1, \beta = 1$, then the system is identified as the D0-$\overline{\rm D0}$ system where D0-brane and $\overline{\rm D0}$-brane sit at $(x^1, x^2) = (0, 0)$ and $(x^1, x^2) = (\pi, \pi)$. For example the open string spectrum at $\alpha = 0$, $\beta = 0$ is shown to be the same as the spectrum at $\alpha = 1, \beta = 1$ because the momentum shift is $\Delta P_{\phi^1} = \pm 1/\sqrt{2}$, $\Delta P_{\phi^2} = \pm 1/\sqrt{2}$ and for each state the value of $(-1)^F h_1 h_2$ does not change. But in order to prove

| sector | $a$ | $b$ | $c$ | $d$ | $\Delta P_{\phi^1}$ | $\Delta P_{\phi^2}$ |
|---|---|---|---|---|---|---|
| $1 \otimes \tau_3$ | 1 | $i$ | 1 | $-i$ | $\alpha/\sqrt{2}$ | $\beta/\sqrt{2}$ |
| $\sigma_1 \otimes \tau_1$ | 1 | $i$ | $-1$ | $i$ | $\alpha/\sqrt{2}$ | $-\beta/\sqrt{2}$ |
| $\sigma_3 \otimes 1$ | 1 | $-i$ | $-1$ | $-i$ | $-\alpha/\sqrt{2}$ | $\beta/\sqrt{2}$ |
| $\sigma_2 \otimes \tau_2$ | 1 | $-i$ | 1 | $i$ | $-\alpha/\sqrt{2}$ | $-\beta/\sqrt{2}$ |

**Table 1:** Momentum shifts due to the tachyon condensation. Here we introduced four coefficients $a, b, c, d$ of a given vertex $V$ as $V = a\,(1\otimes\tau_3) + b\,(\sigma_1\otimes\tau_1) + c\,(\sigma_3\otimes 1) + d\,(\sigma_2\otimes\tau_2)$.

the claim it is necessary to distinguish D0-$\overline{\text{D0}}$ from its T-dual equivalent D2-$\overline{\text{D2}}$ at the self-dual radii $R_1 = R_2 = 1$ and explain[6] the emergence of NSNS and RR-charge corresponding to D0-$\overline{\text{D0}}$. For these purposes the boundary state description which will be discussed in the following sections is very useful and systematic since the D-branes are represented in the closed string Hilbert space in this description.

## 2.2 Tachyon condensation in $\mathbb{T}^4/\mathbb{Z}_2$ orbifold

Let us denote[7] $y^6, y^7, y^8, y^9$ as the coordinates of $\mathbb{T}^4/\mathbb{Z}_2$ with the involution $I_4$ : $(y^6, y^7, y^8, y^9) \rightarrow (-y^6, -y^7, -y^8, -y^9)$ and assume the radii of the torus are given as $R^6 = \sqrt{2}$, $R^7 = R^8 = R^9 = R$.

First we consider a fractional D0-$\overline{\text{D0}}$ system where a D0-brane is sitting on a fixed point $(y^6, y^7, y^8, y^9) = (0, 0, 0, 0)$ and a $\overline{\text{D0}}$-brane is sitting on another fixed point $(y^6, y^7, y^8, y^9) = (\pi\sqrt{2}, 0, 0, 0)$. Each of them has 1/2 tension and 1/2 RR charge of a bulk D0-brane and can be interpreted as a D2-brane wrapped on the vanishing 2-cycle which corresponds to the fixed point [28].

Such a system has no tachyonic modes which survive the $I_4$ projection and therefore is stable at the critical radius. One of the marginal "tachyon" vertex in (0)-picture which represents a tachyon kink in the $y^6$ direction is given as

$$V_T = \psi^6 \left( e^{i\frac{1}{\sqrt{2}}(Y_R^6 - Y_L^6)} \pm e^{-i\frac{1}{\sqrt{2}}(Y_R^6 - Y_L^6)} \right) \otimes \sigma_1 \,, \tag{2.13}$$

where $\pm$ depend on the relative twisted sector charge of the system or equally the projection $I_4 = \pm 1$ and below we only consider the case of $+$ sign. Using the bosonizations and fermionizations (2.5), it is easy to see $V_T \propto \partial\phi^6 \otimes \sigma_1$ and the

[6]For the NSNS-charge the explanation is given in [15] perturbatively by considering a certain disk amplitude where a NSNS vertex operator was inserted.

[7]Here we have used not $x$ but $y$ because later we will identify these coordinates as $\pi/4$ rotated ones.

tachyon condensation[8] is described by the following Wilson line[9]

$$W = \exp\left( i \frac{\alpha}{2\sqrt{2}} \oint \partial\phi^6 \otimes \sigma_1 \right). \tag{2.14}$$

If we condense the tachyon into $\alpha = 1$, then the system is identified with the non-BPS D1-brane stretching between the fixed points. The justification of this statement will be given later by constructing the boundary state. In [8] this non-BPS D1-brane is identified with a D2-brane wrapped on a non-supersymmetric cycle. Later we will also construct the marginal deformation from D4-$\overline{\text{D4}}$ to D0-$\overline{\text{D0}}$ in this orbifold theory.

There is also a known interesting fact. If we consider a non-BPS D1-brane at the special radius $R = 1/\sqrt{2}$, then the vacuum amplitude of the system vanishes and the system develops the bose-fermi degeneracy [26]. Later this phenomenon will be discussed in terms of the boundary state description.

## 3. Boundary state description of tachyon condensation

In this section, we construct the boundary state for a D2-$\overline{\text{D2}}$ system and condensate a tachyon vortex pair. Mainly we follow the line of [24], where a tachyon kink was considered. The crucial difference from that case is the emergence of non-trivial "Chan-Paton factors" in closed string sectors. After the condensation the final object is identified with a D0-$\overline{\text{D0}}$ system as expected. Next we also calculate the vacuum amplitude and investigate the consistency with open string picture. Finally we generalize these results into the higher codimension cases. The definition and brief review of boundary states are given in appendix A.

### 3.1 Bosonization of the boundary state and tachyon condensation

First the boundary state for a D2-brane at $x^i$ $(i = 3 \sim 9)$ which is extended to $x^1, x^2$ without any Wilson lines is given as follows:

$$\begin{aligned}
|\mathrm{D2}, x^i\rangle &= \frac{T_{p=2}}{2}\Big( |\mathrm{D2}, x^i\rangle_{\mathrm{NSNS}} + |\mathrm{D2}, x^i\rangle_{\mathrm{RR}} \Big), \\
|\mathrm{D2}, x^i\rangle_{\mathrm{NSNS}} &= \frac{1}{2}\int \left(\frac{dk}{2\pi}\right)^7 e^{ikx} \Big[ |\mathrm{D2}, +, k_X^i\rangle_{\mathrm{NSNS}} - |\mathrm{D2}, -, k_X^i\rangle_{\mathrm{NSNS}} \Big], \\
|\mathrm{D2}, x^i\rangle_{\mathrm{RR}} &= 2\int \left(\frac{dk}{2\pi}\right)^7 e^{ikx} \Big[ |\mathrm{D2}, +, k_X^i\rangle_{\mathrm{RR}} + |\mathrm{D2}, -, k_X^i\rangle_{\mathrm{RR}} \Big],
\end{aligned} \tag{3.1}$$

[8]Note that this marginal deformation is just the opposite to that considered in [8], where the deformation from non-BPS D1-brane to D0-$\overline{\text{D0}}$ is considered.

[9]If we consider the case of the opposite twisted charge, then we get $V_T \propto \partial\phi'^6 \otimes \sigma_1$ and $W = \exp(i \frac{\alpha}{2\sqrt{2}} \oint \partial_n \phi'^6 \otimes \sigma_1)$, where $\partial_n$ denotes derivative in the normal direction.

where $T_p = 2^{3-p}\pi^{7/2-p}$ is the normalization[10] of the D$p$-brane boundary state and $k^i (i = 3 \sim 9)$ are the momenta in the direction of $x^i$. The explicit forms of $|\mathrm{D2}, \gamma, k^i\rangle_{\mathrm{sector}}$ are given below. The NSNS-sector is

$$
\begin{aligned}
\big|\mathrm{D2}, \gamma, k_X^i\big\rangle_{\mathrm{NSNS}} = \exp&\left[\sum_{n=1}^{\infty}\frac{1}{n}\left\{(\alpha_X)^0_{-n}(\tilde\alpha_X)^0_{-n} + \sum_{j=3}^{7}(\alpha_X)^j_{-n}(\tilde\alpha_X)^j_{-n}\right\}\right]\times \\
&\times \exp\left[-i\,\gamma\sum_{n=1}^{\infty}\left\{\chi^0_{-n+1/2}\,\tilde\chi^0_{-n+1/2} + \sum_{j=3}^{7}\chi^j_{-n+1/2}\tilde\chi^j_{-n+1/2}\right\}\right]\times \\
&\times \exp\left[-\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{1}{n}\,(\alpha_X)^i_{-n}(\tilde\alpha_X)^i_{-n}\right]\times \\
&\times \exp\left[+i\,\gamma\sum_{n=1}^{\infty}\sum_{i=1}^{2}\chi^i_{-n+1/2}\,\tilde\chi^i_{-n+1/2}\right]\times \big|\mathrm{D2}, \gamma, k_X^i\big\rangle^{(0)}_{\mathrm{NSNS}}\,, \qquad (3.2)
\end{aligned}
$$

and zero-mode

$$
\big|\mathrm{D2}, \gamma, k_X^i\big\rangle^{(0)}_{\mathrm{NSNS}} = \sum_{\vec w_X \in \mathbb{Z}^2} |\vec 0, \vec w_X\,\rangle \otimes \big|k_X^i\big\rangle \otimes |\Omega\,\rangle^{(0)}_{\mathrm{NSNS}}\,, \qquad (3.3)
$$

where $|\Omega\,\rangle^{(0)}_{\mathrm{NSNS}}$ is the vacuum of world sheet theory and $|\vec n_X, \vec w_X\,\rangle$ represents the zero mode part of $T^2$ which has momenta $\vec n_X = (n_X^1, n_X^2)$ and windings $\vec w_X = (w_X^1, w_X^2)$. The RR-sector is given as

$$
\begin{aligned}
\big|\mathrm{D2}, \gamma, k_X^i\big\rangle_{\mathrm{RR}} = \exp&\left[\sum_{n=1}^{\infty}\frac{1}{n}\left\{(\alpha_X)^0_{-n}(\tilde\alpha_X)^0_{-n} + \sum_{j=3}^{7}(\alpha_X)^j_{-n}(\tilde\alpha_X)^j_{-n}\right\}\right]\times \\
&\times \exp\left[-i\,\gamma\sum_{n=1}^{\infty}\left\{\chi^0_{-n}\,\tilde\chi^0_{-n} + \sum_{j=3}^{7}\chi^j_{-n}\,\tilde\chi^j_{-n}\right\}\right]\times \\
&\times \exp\left[-\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{1}{n}\,(\alpha_X)^i_{-n}(\tilde\alpha_X)^i_{-n}\right]\times \\
&\times \exp\left[+i\,\gamma\sum_{n=1}^{\infty}\sum_{i=1}^{2}\chi^i_{-n}\,\tilde\chi^i_{-n}\right]\times \big|\mathrm{D2}, \gamma, k_X^i\big\rangle^{(0)}_{\mathrm{RR}}\,, \qquad (3.4)
\end{aligned}
$$

and zero-mode

$$
\big|\mathrm{D2}, \gamma, k_X^i\big\rangle^{(0)}_{\mathrm{RR}} = \sum_{\vec w_X \in \mathbb{Z}^2} |\vec 0, \vec w_X\,\rangle \otimes \big|k_X^i\big\rangle \otimes |\Omega, \gamma\,\rangle^{(0)}_{\mathrm{RR}}\,, \qquad (3.5)
$$

where $|\Omega, \gamma\,\rangle^{(0)}_{\mathrm{RR}}$ is the solution to

$$
\{\chi_0^\mu - i\gamma\tilde\chi_0^\mu\}\;|\Omega, \gamma\,\rangle^{(0)}_{\mathrm{RR}} = \{\chi_0^i + i\gamma\tilde\chi_0^i\}\;|\Omega, \gamma\,\rangle^{(0)}_{\mathrm{RR}} = 0\,, \quad (\mu = 0, 1, 2)\,. \qquad (3.6)
$$

[10]This can be determined by computing the cylinder amplitude.

The normalizations of the zero modes are defined as

$$
\begin{aligned}
\langle\, \Omega \mid \Omega\,\rangle^{(0)}_{\rm NSNS} &= 1\,, & \langle\, \Omega,\gamma \mid \Omega,\gamma'\,\rangle^{(0)}_{\rm RR} &= \delta_{\gamma,\gamma'}\,,\\
\left\langle\, \vec{n}_X, \vec{w}_X \mid \vec{n'}_X, \vec{w'}_X \,\right\rangle &= V\delta_{n,n'}\,\delta_{w,w'}\,, & \langle\, k^i_X \mid k'^i_X\,\rangle &= (2\,\pi)^7\delta^7(k-k')\,,
\end{aligned} \tag{3.7}
$$

where $V$ is the volume of the time direction. Note that this is the solution to the condition (A.4) for a boundary state of a D2-brane. We use $\gamma = \pm 1$ to indicate each choice of the open string boundary condition (A.4). Also note that since we have used the light cone formalism [29, 13], the superscripts of oscillators run from 0 to 7, not to 9. We have divided the oscillator parts into $X^0, X^3 \sim X^7$ and $X^1, X^2$ ($\chi^\mu$ are also divided) in eqs. (3.2) and (3.4). This is because $X^0, X^3 \sim X^7$ part does not contribute to the later calculations of tachyon condensation importantly. Therefore we abbreviate $X^0, X^3 \sim X^7, \chi^0, \chi^3 \sim \chi^7$ part and $k^i$ from now on.

Next, we construct the boundary state for D2-$\overline{\rm D2}$ system where the position of the D2-brane and the $\overline{\rm D2}$-brane is $x^i = 0$. This is given by the superposition of the boundary states for a D2-brane as

$$
\left\{
\begin{aligned}
\left|{\rm D2}-\overline{\rm D2},\gamma\right\rangle_{\rm NSNS} &= |{\rm D2},\gamma\,\rangle_{\rm NSNS} + |{\rm D2}',\gamma\,\rangle_{\rm NSNS}\,,\\
\left|{\rm D2}-\overline{\rm D2},\gamma\right\rangle_{\rm RR} &= |{\rm D2},\gamma\,\rangle_{\rm RR} - |{\rm D2}',\gamma\,\rangle_{\rm RR}\,.
\end{aligned}
\right. \tag{3.8}
$$

Here, we have two important points. One point is that we have switched on $\mathbb{Z}_2$ Wilson lines of the second D2-brane and we have expressed such a boundary state as that with a prime. Next point is the second D2-brane is the anti D-brane. Since an anti D-brane has the opposite RR charge to a D-brane, we have added a minus sign to the second boundary state of RR sector.

From eq. (3.8), the oscillator parts of $|{\rm D2}\text{-}\overline{\rm D2},\gamma\rangle_{\rm NSNS,RR}$ are the same as eqs. (3.2) and (3.4), and the zero mode parts are given by

$$
\begin{aligned}
\left|{\rm D2}-\overline{\rm D2},\gamma\right\rangle^{(0)}_{\rm NSNS} &= \sum_{\vec{w}_X\in\mathbb{Z}^2} |\vec{0},\vec{w}_X\rangle^{(0)}_{\rm NSNS} + (-1)^{w^1_X+w^2_X}|\vec{0},\vec{w}_X\rangle^{(0)}_{\rm NSNS}\,,\\
|{\rm D2}-\overline{\rm D2},\gamma\rangle^{(0)}_{\rm RR} &= \sum_{\vec{w}_X\in\mathbb{Z}^2} |\vec{0},\vec{w}_X,\gamma\rangle^{(0)}_{\rm RR} - (-1)^{w^1_X+w^2_X}|\vec{0},\vec{w}_X,\gamma\rangle^{(0)}_{\rm RR}\,,
\end{aligned} \tag{3.9}
$$

where the phase factors $(-1)^{w^1_X}, (-1)^{w^2_X}$ are due to $\mathbb{Z}_2$ Wilson lines [3, 24].

Then let us perform $\pi/4$ rotation (2.4) and rewrite[11] the boundary state $|{\rm D2}\text{-}\overline{\rm D2},\gamma\rangle_{\rm NSNS,RR}$ by using the fields $(Y^1, Y^2, \psi^1, \psi^2)$ as follows:

[11] Note that here we have regarded the radii of $Y^1, Y^2$ direction as $R^1 = R^2 = 1/\sqrt{2}$ and we have used the relations $w^1_Y = w^1_X + w^2_X,\ \ w^2_Y = w^1_X - w^2_X$.

$$\left|\mathrm{D2}-\overline{\mathrm{D2}},\gamma\right\rangle_{\mathrm{NSNS}} = \exp\left[-\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{1}{n}\,(\alpha_Y)^i_{-n}(\tilde{\alpha}_Y)^i_{-n}\right]\times \tag{3.10}$$

$$\times\ \exp\left[+i\,\gamma\sum_{n=1}^{\infty}\sum_{i=1}^{2}\psi^i_{-n+1/2}\,\tilde{\psi}^i_{-n+1/2}\right]2\sum_{\vec{w}_Y\in\mathbb{Z}^2}|\vec{0},2\vec{w}_Y\rangle^{(0)}_{\mathrm{NSNS}}\,,$$

$$\left|\mathrm{D2}-\overline{\mathrm{D2}},\gamma\right\rangle_{\mathrm{RR}} = \exp\left[-\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{1}{n}(\alpha_Y)^i_{-n}(\tilde{\alpha}_Y)^i_{-n}\right]\times \tag{3.11}$$

$$\times\ \exp\left[+i\,\gamma\sum_{n=1}^{\infty}\sum_{i=1}^{2}\psi^i_{-n}\,\tilde{\psi}^i_{-n}\right]2\sum_{\vec{w}_Y\in\mathbb{Z}^2}\left|\vec{0},2\vec{w}_Y+\vec{1},\gamma\right\rangle^{(0)}_{\mathrm{RR}}\,,$$

where we defined $\vec{1}=(1,1)$.

As explained in section 2, we have to change the base of the world sheet fields $(Y,\psi)$ into the bosonized ones $(\phi,\eta)$ in order to describe the tachyon condensation. Then by using $(\phi,\eta)$ modes how are the boundary states (3.10) and (3.11) represented? Generalizing the discussion in [24], we argue that the following boundary states are equivalent to eqs. (3.10) and (3.11), respectively for $\gamma=+1$:

$$\left|\mathrm{D2}-\overline{\mathrm{D2}},+\right\rangle_{\mathrm{NSNS}} = 2\ \exp\left[-\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{1}{n}\,\phi^i_{-n}\,\tilde{\phi}^i_{-n}\right]\times \tag{3.12}$$

$$\times\ \exp\left[+i\sum_{n=1}^{\infty}\sum_{i=1}^{2}\eta^i_{-n+1/2}\,\tilde{\eta}^i_{-n+1/2}\right]\sum_{\vec{w}_\phi\in\mathbb{Z}^2}|\vec{0},2\vec{w}_\phi\,\rangle^{(0)}_{\mathrm{NSNS}}\,,$$

$$\left|\mathrm{D2}-\overline{\mathrm{D2}},+\right\rangle_{\mathrm{RR}} = 2\ \exp\left[-\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{1}{n}\phi^i_{-n}\tilde{\phi}^i_{-n}\right]\exp\left[+i\sum_{n=1}^{\infty}\sum_{i=1}^{2}\eta^i_{-n}\,\tilde{\eta}^i_{-n}\right]\times$$

$$\times\sum_{\vec{w}_\phi\in\mathbb{Z}^2}\left|\vec{0},2\vec{w}_\phi+\vec{1},+\right\rangle^{(0)}_{\mathrm{RR}}. \tag{3.13}$$

These are obtained simply by replacing $(\alpha_Y)^i_n$, $\psi^i_n$, $w^i_X$ in eqs. (3.10) and (3.11) with $\phi^i_n$, $\eta^i_n$, $w^i_\phi$. In the appendix B we show they indeed satisfy the desirable boundary conditions if we take the detailed (second-type) cocycle factors into consideration:

$$\partial_2 Y^i(w,\bar{w})|_{\sigma_2=0}\left|\mathrm{D2}-\overline{\mathrm{D2}},+\right\rangle_{\mathrm{NSNS,RR}}=0\,, \tag{3.14}$$

$$\left(\psi^i_R(w)-i\,\psi^i_L(\bar{w})\right)|_{\sigma_2=0}\left|\mathrm{D2}-\overline{\mathrm{D2}},+\right\rangle_{\mathrm{NSNS,RR}}=0\,. \tag{3.15}$$

Note that eqs. (3.14) and (3.15) are not enough[12] for the proof of the equivalence. But further we can see that for several closed string states eqs. (3.12) and (3.13) have

[12]This is because these constraints do not determine the detailed structures of the zero modes such as Wilson lines.

the same overlap as eqs. (3.10) and (3.11) do.[13] The third evidence is the equality of their partition functions, which we will see in the next subsection. We propose that these three evidences are enough for the proof of the equivalence.

On the other hand, $|\mathrm{D2}\text{-}\overline{\mathrm{D2}},-\rangle_{\mathrm{NSNS,RR}}$ is given by acting left-moving fermion number operator $(-1)^{\tilde{F}_Y}$ of $(Y,\psi)$ system:

$$\big|\mathrm{D2}-\overline{\mathrm{D2}},-\big\rangle_{\mathrm{NSNS,RR}} = (-1)^{\tilde{F}_Y}\,\big|\mathrm{D2}-\overline{\mathrm{D2}},+\big\rangle_{\mathrm{NSNS,RR}}\,. \tag{3.16}$$

Note that the action of $(-1)^{\tilde{F}_Y}$ is given as

$$\begin{aligned}(-1)^{\tilde{F}_Y}\ :\ (\tilde{\alpha}_Y)^i_n \longrightarrow (\tilde{\alpha}_Y)^i_n\,,\quad \tilde{\psi}^i_n \longrightarrow -\tilde{\psi}^i_n\,,\\ \tilde{\phi}^i_n \longrightarrow -\tilde{\phi}^i_n\,,\quad \tilde{\eta}^i_n \longrightarrow \tilde{\eta}^i_n\,,\end{aligned} \tag{3.17}$$

and to zero mode

$$(-1)^{\tilde{F}_Y}\ :\ |\vec{n}_\phi\,,\,\vec{w}_\phi\,\rangle \longrightarrow (\text{phase}) \times \left|\frac{\vec{w}_\phi}{2}\,,\,2\vec{n}_\phi\right\rangle\,, \tag{3.18}$$

where "phase" comes from cocycle factors. Therefore for example, $|\mathrm{D2}\text{-}\overline{\mathrm{D2}},-\rangle_{\mathrm{NS}}$ is given by

$$\begin{aligned}\big|\mathrm{D2}-\overline{\mathrm{D2}},-\big\rangle_{\mathrm{NSNS}} &= 2\exp\left[+\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{1}{n}\,\phi^i_{-n}\,\tilde{\phi}^i_{-n}\right]\exp\left[+i\sum_{n=1}^{\infty}\sum_{i=1}^{2}\eta^i_{-n+1/2}\,\tilde{\eta}^i_{-n+1/2}\right]\times\\ &\times\sum_{\vec{w}_\phi\in\mathbb{Z}^2}(-1)^{w^1_\phi+w^2_\phi}|\vec{w}_\phi,\vec{0}\rangle^{(0)}_{\mathrm{NSNS}}\,.\end{aligned} \tag{3.19}$$

Then it is straightforward to condense the tachyon by using the Wilson lines (2.9). Since closed strings usually do not have Chan-Paton factors, the tachyon condensation in the closed string viewpoint corresponds to the insertion of the trace of (2.9) in front of the boundary state and the trace is given as

$$W_1(\alpha,\beta) \propto \cos\left(\frac{\pi\alpha\, w^1_\phi}{2}\right)\cos\left(\frac{\pi\beta\, w^2_\phi}{2}\right)\,. \tag{3.20}$$

But, this is not sufficient. In [30] the authors argued that the Wess-Zumino terms in the effective action of D2-$\overline{\mathrm{D2}}$ systems should possess the following coupling.

$$\int C_1\wedge dT\wedge d\bar{T}\,, \tag{3.21}$$

where $T,\bar{T}$ are complex tachyon fields, and $C_1$ is the R-R 1-form which couples to a D0-brane. This implies that $C_1$ have Chan-Paton factor $\sigma_3$ since $T,\bar{T}$ have Chan-Paton factor $\sigma_1 \pm i\,\sigma_2$, respectively. At first sight you may think such an idea is not

[13] This is almost the same calculation as that in [24, appendix B]. Thus we omit its detail.

acceptable, but it is required to reproduce the correct spectrum of open strings and satisfy the Cardy's condition as we will see in the next subsection. Therefore we regard our results as another evidence of such an idea. Similar "Chan-Paton factor for closed string vertex" was discussed [7, 1] in the case of a non-BPS D-brane,[14] where it was argued that the branch cut due to a RR-vertex provides an extra Chan-Paton factor if its one end is on the non-BPS D-brane. We also argue that some of the states in NSNS sector have the Chan-Paton factor $\sigma_3$. This fact can also be verified by the Cardy's condition and will be required due to the supersymmetry of the bulk theory. Therefore in these sectors we should insert $\sigma_3$ in the trace.

Then the tachyon condensation switches not only (3.20) but also

$$W_{\sigma_3}(\alpha,\beta) \propto \sin\left(\frac{\pi\alpha\, w_\phi^1}{2}\right)\sin\left(\frac{\pi\beta\, w_\phi^2}{2}\right), \tag{3.22}$$

which is obtained by inserting $\sigma_3$ in the trace.

Then by switching both contributions we obtain the following boundary state[15]

$$\begin{aligned} |B(\alpha,\beta),+\rangle_{\mathrm{NSNS}} = {} & 2\,\exp\left[-\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{1}{n}\,\phi^i_{-n}\,\tilde{\phi}^i_n\right]\exp\left[+i\sum_{n=1}^{\infty}\sum_{i=1}^{2}\eta^i_{-n+1/2}\,\tilde{\eta}^i_{-n+1/2}\right]\times \\ & \times\sum_{\vec{w}_\phi\in\mathbb{Z}^2}\Big[\cos\left(\pi\alpha w_\phi^1\right)\cos\left(\pi\beta w_\phi^2\right)+\sin\left(\pi\alpha\, w_\phi^1\right)\sin\left(\pi\beta\, w_\phi^2\right)\Big]\times \\ & \times|\vec{0},2\vec{w}_\phi\rangle^{(0)}_{\mathrm{NSNS}}, \end{aligned} \tag{3.23}$$

$$\begin{aligned} |B(\alpha,\beta),+\rangle_{\mathrm{RR}} = {} & 2\,\exp\left[-\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{1}{n}\,\phi^i_{-n}\,\tilde{\phi}^i_n\right]\exp\left[+i\sum_{n=1}^{\infty}\sum_{i=1}^{2}\eta^i_{-n}\,\tilde{\eta}^i_{-n}\right]\times \\ & \times\sum_{\vec{w}_\phi\in\mathbb{Z}^2}\Bigg[\cos\left\{\pi\alpha\left(w_\phi^1+\frac{1}{2}\right)\right\}\cos\left\{\pi\beta\left(w_\phi^2+\frac{1}{2}\right)\right\}+ \\ & \qquad + \sin\left\{\pi\alpha\left(w_\phi^1+\frac{1}{2}\right)\right\}\sin\left\{\pi\beta\left(w_\phi^2+\frac{1}{2}\right)\right\}\Bigg]\times \\ & \times\left|\vec{0},2\vec{w}_\phi+\vec{1},+\right\rangle^{(0)}_{\mathrm{RR}}. \end{aligned} \tag{3.24}$$

This satisfies $|B(0,0),+\rangle_{\mathrm{NSNS,RR}} = |\mathrm{D2}\text{-}\overline{\mathrm{D2}},+\rangle_{\mathrm{NSNS,RR}}$.

As explained in section 2.1 the point $\alpha=\beta=1$ is expected to be identified as a D0-$\overline{\mathrm{D0}}$ system [15] where a D0-brane and a $\overline{\mathrm{D0}}$-brane are produced at $(x_1,x_2)=$

[14] For example, in the case of the non-BPS D2-brane the Wess-Zumino term is written as $\int C_1\wedge\, dT$ [31].

[15] Strictly speaking, from the above explanation we cannot decide the relative normalization between the first and the second term. This is determined by the vacuum energy calculation in the next subsection.

$(0,0)$ , $(\pi,\pi)$, respectively. The boundary state of this D0-$\overline{\text{D0}}$ system is

$$\big|\text{D0}-\overline{\text{D0}},+\big\rangle_{\text{NSNS}} = 2\,\exp\left[\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{1}{n}(\alpha_Y)^i_{-n}(\tilde{\alpha}_Y)^i_{-n}\right]\times$$
$$\times\,\exp\left[-i\sum_{n=1}^{\infty}\sum_{i=1}^{2}\psi^i_{-n+\frac{1}{2}}\tilde{\psi}^i_{-n+\frac{1}{2}}\right]\sum_{\vec{n}_Y\in\mathbb{Z}^2}|\vec{n}_Y,\vec{0}\rangle^{(0)}_{\text{NSNS}}\,, \tag{3.25}$$

$$\big|\text{D0}-\overline{\text{D0}},+\big\rangle_{\text{RR}} = 2\,\exp\left[\sum_{n=1}^{\infty}\sum_{i=1}^{2}\frac{1}{n}\,(\alpha_Y)^i_{-n}(\tilde{\alpha}_Y)^i_{-n}\right]\exp\left[-i\sum_{n=1}^{\infty}\sum_{i=1}^{2}\psi^i_{-n}\,\tilde{\psi}^i_{-n}\right]\times$$
$$\times\sum_{\vec{n}_Y\in\mathbb{Z}^2}\left|\vec{n}_Y+\frac{\vec{1}}{2},\vec{0},+\right\rangle^{(0)}_{\text{RR}}. \tag{3.26}$$

On the other hand, at this point the zero mode parts of eqs. (3.23) and (3.24) become, respectively

$$\sum_{\vec{w}_\phi\in\mathbb{Z}^2}(-1)^{w^1_\phi+w^2_\phi}|\vec{0},2\vec{w}_\phi\rangle^{(0)}_{\text{NSNS}}\,,\qquad \sum_{\vec{w}_\phi\in\mathbb{Z}^2}(-1)^{w^1_\phi+w^2_\phi}|\vec{0},2\vec{w}_\phi+\vec{1}\rangle^{(0)}_{\text{RR}}\,. \tag{3.27}$$

Then just as we have verified that eqs. (3.12) and (3.13) are equivalent to eqs. (3.10) and (3.11), we can also verify that $|B(1,1),+\rangle_{\text{NSNS,RR}}$ is equivalent to eqs. (3.25) and (3.26) in the same way. For example, eqs. (3.25) and (3.26) indeed satisfy the following equations which represent the boundary conditions of D0-branes:

$$\partial_1 Y^i(w)|_{\sigma_2=0}\,|B(1,1),+\rangle_{\text{NSNS,RR}} = 0\,, \tag{3.28}$$
$$\big(\psi^i_R(w)+i\,\psi^i_L(\bar{w})\big)\,|_{\sigma_2=0}\,|B(1,1),+\rangle_{\text{NSNS,RR}} = 0\,,\quad (i=1,2)\,. \tag{3.29}$$

In other words the tachyon condensation from $\alpha=\beta=0$ to $\alpha=\beta=1$ changes the boundary conditions (3.14) and (3.15) into (3.28) and (3.29) and the crucial difference between them is that the latter has the phase factor $(-1)^{w^1_\phi+w^2_\phi}$. At $\alpha=\beta=0$ only the first term of eq. (3.24) is non zero and this corresponds to the RR charge of D2-brane. As the tachyon is condensed the second term also ceases to be zero and this means[16] that the RR charge of the D0-brane is generated. Finally at $\alpha=\beta=1$ only the second term is non zero and this is the pure D0-brane RR charge. Note that if we ignored the factor (3.22) which corresponds to $\sigma_3$ sector, then the RR-sector boundary state would vanish at $\alpha=\beta=1$ and be inconsistent. In this way we see explicitly in the closed string formalism that a tachyon kink on a brane-antibrane system produces a codimension two D-brane (see figure 1).

[16] It is easy to see that if $\alpha,\beta$ are small, then the second term is proportional to $V_{T1}V_{T2}|\text{D2}\rangle_{\text{RR}}\sim|\text{D0}\rangle_{\text{RR}}$.

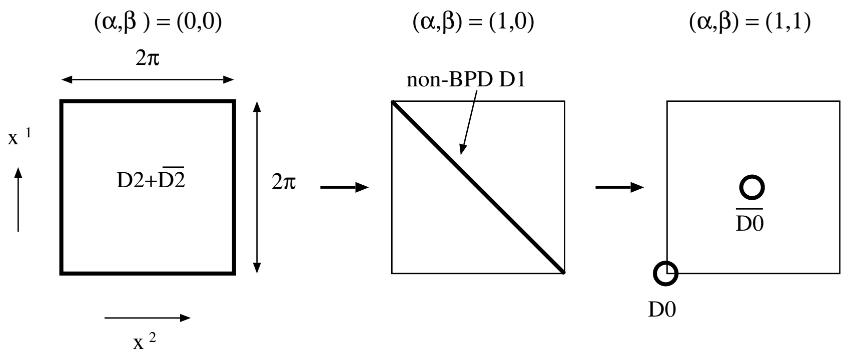


**Figure 1:** The tachyon condensation in D2-$\overline{\text{D2}}$ system.

Let us turn to the other points of $\alpha, \beta$. It is easy to see that at $(\alpha, \beta) = (0,1), (1,0)$ the RR-sector boundary state does vanish and each system corresponds to a non-BPS D1-brane stretching along the direction of $Y^1$ or $Y^2$, respectively (see figure 1). Physically this can be interpreted as the statement that a tachyon kink produces a codimension one (non-BPS) D-brane. All of these identifications will be verified further by the calculation of vacuum amplitudes including the detailed normalization.

### 3.2 Calculation of the vacuum amplitude

Here we calculate the vacuum amplitude of D2-$\overline{\text{D2}}$ system for every value of $\alpha, \beta$ and translate it from the viewpoint of open string. As a result it will be shown that the boundary state have the correct normalizations or equally correct NSNS and RR-charge needed for the identification and that the additional NSNS and RR sector discussed before are indeed required in order to satisfy the Cardy's condition.

First let us define the propagator for closed string as

$$\Delta = \frac{1}{2} \int_0^\infty ds\; e^{-sH_c} \sim \frac{1}{k^2} + \cdots\,, \tag{3.30}$$

where $H_c$ denotes the closed string hamiltonian and its explicit form is given as

$$\begin{aligned} H_c = {} & \sum_{i=1,2} \left( \frac{(n^i_\phi)^2}{2R_i^2} + \frac{1}{2}\,(w^i_\phi)^2 R_i^2 \right) + \\ & + \sum_{i=1,2} \left\{ \sum_n \left( \phi^i_{-n}\phi^i_n + \tilde{\phi}^i_{-n}\tilde{\phi}^i_n \right) + \sum_r \left( \eta^i_{-r}\eta^i_r + \tilde{\eta}^i_{-r}\tilde{\eta}^i_r \right) \right\} + \\ & + \sum_{i=3}^{9} \frac{1}{2}\,(k^i)^2 + \sum_{i=0,3}^{7} \left\{ \sum_n \left( \alpha^i_{-n}\,\alpha^i_n + \tilde{\alpha}^i_{-n}\,\tilde{\alpha}^i_n \right) + \sum_r \left( \chi^i_{-r}\,\chi^i_r + \tilde{\chi}^i_{-r}\,\tilde{\chi}^i_r \right) \right\} + a\,, \end{aligned} \tag{3.31}$$

where $a$ denotes the zero-energy for each sector and is given as $a = -1$ for NSNS-sector and $a = 0$ for RR-sector.

Then the vacuum amplitudes for NSNS- and RR-sector are

$$
\begin{aligned}
Z_{\mathrm{NSNS}} &= \frac{(T_{p=2})^2}{16}\int_0^\infty ds \int \left(\frac{dk}{2\pi}\right)^7 \left(\frac{dk'}{2\pi}\right)^7 \times \\
&\qquad \times \Big\{ \Big\langle B(\alpha,\beta),+,k\,|\Delta|\,B(\alpha,\beta),+,k' \Big\rangle_{\mathrm{NSNS}} - \\
&\qquad\qquad - \langle B(\alpha,\beta),+,k\,|\Delta|\,B(\alpha,\beta),-,k'\rangle_{\mathrm{NSNS}} \Big\} \\
&= \frac{(T_{p=2})^2\, V_{\mathrm{D2}}}{4} \times \\
&\quad \times \int_0^\infty \frac{ds}{(2\pi s)^{7/2}} \Bigg[ \sum_{w_\phi^1, w_\phi^2} \Big\{ \cos^2\left(\pi\alpha\, w_\phi^1\right)\cos^2\left(\pi\beta\, w_\phi^2\right) + \\
&\qquad\qquad + \sin^2\left(\pi\alpha\, w_\phi^1\right)\sin^2\left(\pi\beta\, w_\phi^2\right) \Big\} \times \\
&\qquad\qquad \times q^{(w_\phi^1)^2 + (w_\phi^2)^2} \frac{f_3(q)^8}{f_1(q)^8} - 2\,\frac{f_4(q)^6 f_3(q)^2}{f_1(q)^6 f_2(q)^2} \Bigg], \\
Z_{\mathrm{RR}} &= -(T_{p=2})^2 \int_0^\infty ds \int \left(\frac{dk}{2\pi}\right)^7 \left(\frac{dk'}{2\pi}\right)^7 \Big\langle B(\alpha,\beta),+,k\,|\Delta|\,B(\alpha,\beta),+,k' \Big\rangle_{\mathrm{RR}}, \\
&= -\frac{(T_{p=2})^2\, V_{\mathrm{D2}}}{4} \times \\
&\quad \times \int_0^\infty \frac{ds}{(2\pi s)^{7/2}} \Bigg[ \sum_{w_\phi^1, w_\phi^2} \Bigg\{ \cos^2\left(\pi\alpha\left(w_\phi^1 + \frac{1}{2}\right)\right)\cos^2\left(\pi\beta\left(w_\phi^2 + \frac{1}{2}\right)\right) + \\
&\qquad\qquad + \sin^2\left(\pi\alpha\left(w_\phi^1 + \frac{1}{2}\right)\right)\sin^2\left(\pi\beta\left(w_\phi^2 + \frac{1}{2}\right)\right) \Bigg\} \times \\
&\qquad\qquad \times q^{(w_\phi^1 + 1/2)^2 + (w_\phi^2 + 1/2)^2} \frac{f_2(q)^8}{f_1(q)^8} \Bigg],
\end{aligned} \tag{3.32}
$$

where $V_{\mathrm{D2}} = (2\,\pi)^2 V$ is the volume of D2-brane and we defined

$$
\begin{aligned}
f_1(q) &= q^{1/12}\prod_{n=1}^{\infty}\left(1 - q^{2n}\right), \qquad & f_2(q) &= \sqrt{2}\, q^{1/12}\prod_{n=1}^{\infty}\left(1 + q^{2n}\right), \\
f_3(q) &= q^{-1/24}\prod_{n=1}^{\infty}\left(1 + q^{2n-1}\right), \qquad & f_4(q) &= q^{-1/24}\prod_{n=1}^{\infty}\left(1 - q^{2n-1}\right),
\end{aligned} \tag{3.33}
$$

with $q = e^{-s}$.

Next let us perform modular transformations and interpret this as the open string cylinder amplitude. We define the modulus of the cylinder as $t = \pi/s$ and introduce $\tilde{q} = e^{-\pi t}$. Then we get the following open string amplitude

$$
\begin{aligned}
Z_{\rm NS}^{\rm open} &= 2^{-3/2}\,\pi^{-1} V \times \\
&\times \int_0^\infty dt\; t^{-3/2} \sum_{n_1,n_2} \left[ \frac{1}{2}\left( \tilde{q}^{n_1^2+n_2^2} + \tilde{q}^{(n_1-\alpha)^2+(n_2-\beta)^2)} \right) \frac{f_3(\tilde{q})^8}{f_1(\tilde{q})^8} - \right. \\
&\qquad\qquad \left. - \frac{1}{2}(-1)^{n_1+n_2}\left( \tilde{q}^{n_1^2+n_2^2} + \tilde{q}^{(n_1-\alpha)^2+(n_2-\beta)^2} \right) \frac{f_4(\tilde{q})^8}{f_1(\tilde{q})^8} \right], \\
Z_{R}^{\rm open} &= -2^{-3/2}\,\pi^{-1} V \int_0^\infty dt\; t^{-3/2} \sum_{n_1,n_2} \tilde{q}^{n_1^2+n_2^2} \frac{f_2(\tilde{q})^8}{f_1(\tilde{q})^8}\,,
\end{aligned}
\tag{3.34}
$$

where we used the following identities

$$
\begin{aligned}
\sum_n q^{n^2} &= f_1(q)\, f_3(q)^2\,, \qquad \sum_n (-1)^n\, q^{n^2} = f_1(q)\, f_4(q)^2\,, \\
\sum_n q^{(n-1/2)^2} &= f_1(q)\, f_2(q)^2\,, \qquad f_2(q)\, f_3(q)\, f_4(q) = \sqrt{2}\,,
\end{aligned}
\tag{3.35}
$$

and the modular properties

$$
\begin{aligned}
f_1\left(e^{-\pi/t}\right) &= \sqrt{t}\, f_1\left(e^{-\pi t}\right), \qquad f_2\left(e^{-\pi/t}\right) = f_4\left(e^{-\pi t}\right), \\
f_3\left(e^{-\pi/t}\right) &= f_3\left(e^{-\pi t}\right), \qquad f_4\left(e^{-\pi/t}\right) = f_2\left(e^{-\pi t}\right).
\end{aligned}
\tag{3.36}
$$

Now it is obvious that for each value of $\alpha, \beta$ the open string spectrum is well defined only if we incorporate the additional sector of the boundary state defined in the previous subsection, otherwise the number of open string states for given $n_1, n_2, H_c$ would be fractional. This fact will be more clear if we note that this amplitude can be rewritten as

$$
\begin{aligned}
Z &= Z_{\rm NS}^{\rm open} + Z_{R}^{\rm open}\,, \\
&= V_{\rm D2} \int_0^\infty \frac{dt}{t}\, {\rm Tr}_{{\rm NS}-R} \left[ \frac{1 + (-1)^{F^\phi} h_1^\phi\, h_2^\phi}{2}\, \tilde{q}^{-2H_o} \right],
\end{aligned}
\tag{3.37}
$$

where Tr is the trace over the open string Hilbert space including zero-modes and Chan-Paton sectors. Also $H_o$ means the open string hamiltonian and is given as follows:

$$
\begin{aligned}
H_o = (p^0)^2 &+ \sum_{i=1,2} {R_i}^2 (w_\phi^i)^2 + \sum_{i=1,2} \left\{ \sum_n \phi_{-n}^i\, \phi_n^i + \sum_r \eta_{-r}^i\, \eta_r^i \right\} + \\
&+ \sum_{i=0,3}^{7} \left\{ \sum_n \alpha_{-n}^i\, \alpha_n^i + \sum_r \chi_{-r}^i\, \chi_r^i \right\} + a\,,
\end{aligned}
\tag{3.38}
$$

where $a$ denotes the zero-energy and is given as $a = -1/2$ for NS-sector and $a = 0$ for R-sector.

This physically important constraint (3.37) is generally called Cardy's condition [23]. Also notice that the above open string spectrum is consistent with the momentum shift shown in table 1.

Finally let us verify the identification at particular $\alpha, \beta$. In the case of $(\alpha, \beta) = (1, 0)$ or $(0, 1)$ we get after the modular transformations

$$
\begin{aligned}
Z_{\alpha=1,\beta=0} = Z_{\alpha=0,\beta=1} &= 2^{-3/2}\,\pi^{-1}\,V \int_0^\infty dt\; t^{-3/2} \sum_{n_1,n_2} \tilde{q}^{n_1^2+n_2^2}\, \frac{f_3(\tilde{q})^8 - f_2(\tilde{q})^8}{f_1(\tilde{q})^8}\,, \\
&= 2\,\pi\sqrt{2}\,V \int_0^\infty \frac{dt}{t}\, \mathrm{Tr}_{\mathrm{NS}-R}\left(\tilde{q}^{-2H_o}\right). \qquad (3.39)
\end{aligned}
$$

Therefore we can identify the system as a non-BPS D1-brane of which length is $2\sqrt{2}\,\pi$ as expected. Another case is $(\alpha, \beta) = (1, 1)$ and the amplitude can be written as

$$
\begin{aligned}
Z_{\alpha=1,\beta=1} &= 2^{-3/2}\,\pi^{-1} V \int_0^\infty dt\; t^{-3/2} \sum_{n_1,n_2} \Bigg[ \tilde{q}^{n_1^2+n_2^2}\, \frac{f_3(\tilde{q})^8 - f_2(\tilde{q})^8}{f_1(\tilde{q})^8} - \\
&\qquad - (-1)^{n_1+n_2}\, \tilde{q}^{n_1^2+n_2^2}\, \frac{f_4(\tilde{q})^8}{f_1(\tilde{q})^8} \Bigg], \\
&= 2V \int_0^\infty \frac{dt}{t} \frac{1}{\sqrt{8\pi^2 t}} \sum_{m_1,m_2} \Bigg[ \tilde{q}^{2m_1^2+2m_2^2}\, \frac{f_3(\tilde{q})^8 - f_4(\tilde{q})^8 - f_2(\tilde{q})^8}{2 f_1(\tilde{q})^8} + \\
&\qquad + \tilde{q}^{2(m_1+1/2)^2+2(m_2+1/2)^2}\, \frac{f_3(\tilde{q})^8 + f_4(\tilde{q})^8 - f_2(\tilde{q})^8}{2 f_1(\tilde{q})^8} \Bigg], \\
&= 2V \int_0^\infty \frac{dt}{t}\, \mathrm{Tr}_{\mathrm{NS}-R} \left[ \frac{1+(-1)^F}{2}\, \tilde{q}^{-2H_c} \right] + \\
&\quad + 2V \int_0^\infty \frac{dt}{t}\, \mathrm{Tr}_{\mathrm{NS}-R} \left[ \frac{1-(-1)^F}{2} \tilde{q}^{-2H_c} \right], \qquad (3.40)
\end{aligned}
$$

where we have defined $m_1 = \frac{n_1+n_2}{2}$, $m_2 = \frac{n_1-n_2}{2}$. This shows explicitly that the system is equivalent to a D0-brane and an anti D0-brane which are separated from each other by $\Delta x_1 = \Delta x_2 = \pi$.

### 3.3 Moduli space of non-supersymmetric D-branes

So far we have discussed the boundary states which describe various tachyon condensations in D2-$\overline{\mathrm{D2}}$ system at the critical radii. The tachyon condensations are parameterized by $\alpha, \beta$ which have periodicity $\alpha \sim \alpha + 2, \beta \sim \beta + 2$. At this particular radius the non-supersymmetric D-brane configurations for all values of $\alpha, \beta$ are realized in conformal invariant manners. Figure 2 shows the moduli space of the non-supersymmetric D-brane configurations. In particular $(\alpha, \beta) = (0, 0) \to (1, 0) \to (1, 1)$ can be regarded as a continuous version of the descent relation [9] (see figure 1).

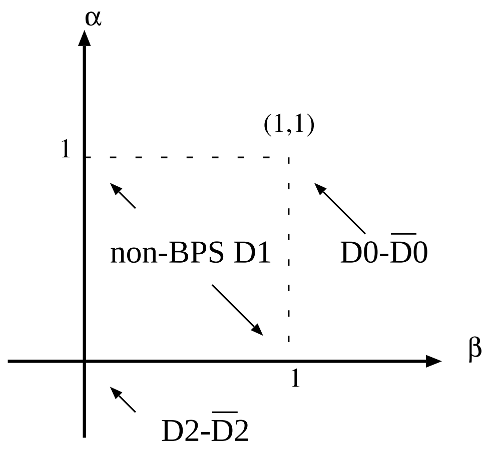


**Figure 2:** The moduli space of the non-supersymmetric D-brane configurations.

Realistically we are interested in the D2-$\overline{\text{D2}}$ system at generic radii. If we shift the radius, tadpoles develop as can be seen by the method discussed in [5, 9, 15] or by computing one point functions using the boundary state we constructed. The tadpoles only vanish at $\sin(\pi\alpha) = \sin(\pi\beta) = 0$, which correspond to D2-$\overline{\text{D2}}$, non-BPS D1-brane and D0-$\overline{\text{D0}}$.

### 3.4 Tachyon condensation in general D$p$-$\overline{\text{D}p}$ systems

In this subsection, we generalize our construction to the higher codimension cases. It is enough to consider the D8-$\overline{\text{D8}}$ system (In odd codimension case, we have only to consider the decay to the D$(-1)$-$\overline{\text{D}(-1)}$ system.). The different points from the analysis of D2-$\overline{\text{D2}}$ system are the slightly complicated choice of the gamma matrices and Chan-Paton factors. We mainly concentrate on these issues in the presentation here.

In particular, we deal with the case where $2^3$ D8-$\overline{\text{D8}}$ pairs at critical radii ($R_1 = \cdots = R_8 = 1$) become $2^3$ D0-$\overline{\text{D0}}$ pairs via the tachyon condensation as the marginal deformation. These $2^3$ soliton-antisoliton pairs represent $2^3$ D0-$\overline{\text{D0}}$ pairs, and a single D0 brane is identified with the single codimension 8 soliton on the D8-$\overline{\text{D8}}$ pair at the point where the tachyon condensation is maximum. Many other points are identified with the bound states of D8-$\overline{\text{D8}}$, D6-$\overline{\text{D6}}$, D4-$\overline{\text{D4}}$, D2-$\overline{\text{D2}}$, D0-$\overline{\text{D0}}$ at critical radii. We show the emergence of the RR charges of the lower-dimensional D-branes explicitly.

We switched on $\mathbb{Z}_2$ Wilson line to the second anti-brane using the Wilson lines [15]

$$\begin{array}{lll} X^1X^2 & \text{direction} & \sigma_3 \otimes \sigma_3 \otimes 1 \ \otimes 1\,, \\ X^3X^4 & \text{direction} & \sigma_3 \otimes 1 \ \otimes 1 \ \otimes 1\,, \\ X^5X^6 & \text{direction} & 1 \ \otimes \sigma_3 \otimes \sigma_3 \otimes 1\,, \\ X^7X^8 & \text{direction} & 1 \ \otimes 1 \ \otimes \sigma_3 \otimes \sigma_3\,. \end{array} \qquad (3.41)$$

where we adopt the following representation of the SO(8) Clifford algebra

$$
\begin{aligned}
\Gamma_1 &= 1 \otimes \sigma_1 \otimes \sigma_2 \otimes \sigma_2 \,, & \Gamma_2 &= \sigma_3 \otimes \sigma_2 \otimes \sigma_2 \otimes \sigma_2 \,, \\
\Gamma_3 &= \sigma_1 \otimes \sigma_2 \otimes \sigma_2 \otimes \sigma_2 \,, & \Gamma_4 &= \sigma_2 \otimes \sigma_2 \otimes \sigma_2 \otimes \sigma_2 \,, \\
\Gamma_5 &= 1 \otimes \sigma_3 \otimes \sigma_2 \otimes \sigma_2 \,, & \Gamma_6 &= 1 \otimes 1 \otimes \sigma_1 \otimes \sigma_2 \,, \\
\Gamma_7 &= 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_2 \,, & \Gamma_8 &= 1 \otimes 1 \otimes 1 \otimes \sigma_1 \,.
\end{aligned} \tag{3.42}
$$

Above matrices for the Wilson lines have the properties such that the matrix in the $X^{2k+1}X^{2k+2}$ direction anticommutes with $\Gamma_{2k+1}, \Gamma_{2k+2}$, and commutes with the other six gamma matrices. Since we have seen that the oscillator parts don't contribute crucially the tachyon condensation, we omit these parts. The zero mode part of the boundary states of this D8-$\overline{\text{D8}}$ system is given by

$$
\begin{aligned}
\big|\text{D8} - \overline{\text{D8}}, \gamma\big\rangle^{(0)}_{\text{NSNS}} = \sum_{\vec{w}_X \in \mathbb{Z}^8} & \Big(1 + (-1)^{w_X^1 + w_X^2}\Big) \Big(1 + (-1)^{w_X^3 + w_X^4}\Big) \times \\
& \times \Big(1 + (-1)^{w_X^5 + w_X^6}\Big) \Big(1 + (-1)^{w_X^7 + w_X^8}\Big) |\vec{0}, \vec{w}_X\rangle^{(0)}_{\text{NSNS}} \,,
\end{aligned} \tag{3.43}
$$

$$
\begin{aligned}
\big|\text{D8} - \overline{\text{D8}}, \gamma\big\rangle^{(0)}_{\text{RR}} = \sum_{\vec{w}_X \in \mathbb{Z}^8} & \Big(1 - (-1)^{w_X^1 + w_X^2}\Big) \Big(1 - (-1)^{w_X^3 + w_X^4}\Big) \times \\
& \times \Big(1 - (-1)^{w_X^5 + w_X^6}\Big) \Big(1 - (-1)^{w_X^7 + w_X^8}\Big) |\vec{0}, \vec{w}_X, \gamma\rangle^{(0)}_{\text{RR}} \,.
\end{aligned} \tag{3.44}
$$

Now we change the variables as follows:

$$
\begin{aligned}
Y^{2k+1} &= \frac{1}{\sqrt{2}} \left(X^{2k+1} + X^{2k+2}\right), & Y^{2k+2} &= \frac{1}{\sqrt{2}} \left(X^{2k+1} - X^{2k+2}\right), \\
\psi^{2k+1}_{R,L} &= \frac{1}{\sqrt{2}} \left(\chi^{2k+1}_{R,L} + \chi^{2k+2}_{R,L}\right), & \psi^{2k+2}_{R,L} &= \frac{1}{\sqrt{2}} \left(\chi^{2k+1}_{R,L} - \chi^{2k+2}_{R,L}\right),
\end{aligned}
$$

where $k = 0, 1, 2, 3$. In terms of these variables, the zero mode parts of the boundary state for D8-$\overline{\text{D8}}$ system are rewritten as follows:

$$
\big|\text{D8} - \overline{\text{D8}}, \gamma\big\rangle^{(0)}_{\text{NSNS}} = 16 \sum_{\vec{w}_Y \in \mathbb{Z}^8} |\vec{0}, 2\vec{w}_Y\rangle^{(0)}_{\text{NSNS}} \,, \tag{3.45}
$$

$$
\big|\text{D8} - \overline{\text{D8}}, \gamma\big\rangle^{(0)}_{\text{RR}} = 16 \sum_{\vec{w}_Y \in \mathbb{Z}^8} \Big|\vec{0}, 2\vec{w}_Y + \vec{1}, \gamma\Big\rangle^{(0)}_{\text{RR}} \,. \tag{3.46}
$$

where $w_Y^{2k+1} = w_X^{2k+1} + w_X^{2k+2}$, $w_Y^{2k+2} = w_X^{2k+1} - w_X^{2k+2}$.

In order to describe the effect of the tachyon condensation, we change the variables using bosonization techniques. First, by fermionization $Y^i$ are represented by fermions $\xi^i, \eta^i$ and next, by bosonization we introduce free bosons $\phi^i$. In the following we neglect the cocycle factors, but we can take their roles into account giving gamma matrix $\widetilde{\Gamma}_i$ to $e^{i\sqrt{2}Y^i}$, and $\widetilde{\Gamma}_{12345678}$ to $\psi^i, \eta^i$ etc. Then the D8-$\overline{\text{D8}}$ system at critical radii is described with the following projection as in section 2.1

$$
(-1)^F h_1 \cdots h_8 = (-1)^{F^\phi} h_1^\phi \cdots h_8^\phi = 1 \,. \tag{3.47}
$$



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Using $(\phi,\eta)$ modes, we can write down the boundary states of D8-$\overline{\text{D8}}$ system for $\gamma=+1$ and their zero modes are given by

$$\big|\mathrm{D8}-\overline{\mathrm{D8}},+\big\rangle^{(0)}_{\mathrm{NSNS}} = 16\sum_{\vec{w}_\phi\in\mathbb{Z}^8} |\vec{0},2\vec{w}_\phi\rangle^{(0)}_{\mathrm{NSNS}}\,, \tag{3.48}$$

$$\big|\mathrm{D8}-\overline{\mathrm{D8}},+\big\rangle^{(0)}_{\mathrm{RR}} = 16\sum_{\vec{w}_\phi\in\mathbb{Z}^8} \Big|\vec{0},2\vec{w}_\phi+\vec{1},+\Big\rangle^{(0)}_{\mathrm{RR}}\,. \tag{3.49}$$

The equivalence of two states written with different variables is provided by the fact that these states satisfy the definition equation of the boundary state for D8-$\overline{\text{D8}}$ system. Also, the boundary states for $\gamma=-1$ is given as in the previous subsection.

Now we are ready to condense the tachyon. The tachyon condensation is represented as the insertion of the following Wilson line [15]

$$\exp\left[\sum_{i=1}^{8}\frac{i\alpha_i}{2\sqrt{2}}\oint\partial\phi^i\otimes\Gamma_i\right], \tag{3.50}$$

where $\alpha_i$ represent the parameters of the condensation and have the periodicity $\alpha_i\sim\alpha_i+2$. This represents the marginal deformation at the critical radius. These traces with the insertion of the various Chan-Paton factors are given as the following five types

$$\begin{aligned}
W_1(\{\alpha_i\}) \;&:\; \prod_{i=1}^{8}\cos\left(\frac{\pi\alpha_i\,w^i_\phi}{2}\right),\\
W_{\Gamma_{ij}}(\{\alpha_i\}) \;&:\; \left[\prod_{i=1}^{6}\cos\left(\frac{\pi\alpha_i\,w^i_\phi}{2}\right)\right]\sin\left(\frac{\pi\alpha_7 w^7_\phi}{2}\right)\sin\left(\frac{\pi\alpha_8 w^8_\phi}{2}\right),\\
{}_8\mathrm{C}_2 \;&=\; 28\,\text{combinations}\,,\\
W_{\Gamma_{ijk\ell}}(\{\alpha_i\}) \;&:\; \left[\prod_{i=1}^{4}\cos\left(\frac{\pi\alpha_i\,w^i_\phi}{2}\right)\right]\left[\prod_{j=5}^{8}\sin\left(\frac{\pi\alpha_j\,w^j_\phi}{2}\right)\right],\\
{}_8\mathrm{C}_4 \;&=\; 70\,\text{combinations}\,,\\
W_{\Gamma_{ijk\ell mn}}(\{\alpha_i\}) \;&:\; \cos\left(\frac{\pi\alpha_1\,w^1_\phi}{2}\right)\cos\left(\frac{\pi\alpha_2\,w^2_\phi}{2}\right)\left[\prod_{i=3}^{8}\sin\left(\frac{\pi\alpha_i\,w^i_\phi}{2}\right)\right],\\
{}_8\mathrm{C}_2 \;&=\; 28\,\text{combinations}\,,\\
W_{\Gamma_{12345678}}(\{\alpha_i\}) \;&:\; \prod_{i=1}^{8}\sin\left(\frac{\pi\alpha_i\,w^i_\phi}{2}\right).
\end{aligned} \tag{3.51}$$

Again the insertion these operators in front of the boundary state corresponds to the tachyon condensation in the closed string sector.

This can be understood from the following Wess-Zumino coupling [30]

$$\int_{\mathrm{D8}-\overline{\mathrm{D8}}} C_{\mathrm{RR}}\wedge\mathrm{STr}\;e^{\mathcal{F}}\,,\qquad \mathcal{F}=\begin{pmatrix} F^+-T\bar{T} & DT\\ D\bar{T} & F^--\bar{T}T\end{pmatrix}, \tag{3.52}$$

where $D = d + A^+ - A^-$ and $T, \bar{T}$ represent the complex tachyon field. The fields $A^+, A^-$ denote the gauge fields on the brane, anti-brane, respectively, which is 0 in this case. Combining with the fact that the tachyon configuration is given by [17, 15]

$$\text{for example} \qquad T(x) \sim \Gamma_i\, x_i\,, \quad \text{at } x_i \sim 0\,, \tag{3.53}$$

we can speculate that the RR fields $C_{2k+1}$ should have the following Chan-Paton factors

$$C_7 : \Gamma_{ij}\,, \qquad C_5 : \Gamma_{ijk\ell}\,, \qquad C_3 : \Gamma_{ijk\ell mn}\,, \qquad C_1 : \Gamma_{12345678}\,. \tag{3.54}$$

For example, this can be understood as the following expression

$$\int_{\mathrm{D8}-\overline{\mathrm{D8}}} C_1\, dx^1 \wedge dx^2 \wedge \ldots \wedge dx^8\, \Gamma_{12345678}\,. \tag{3.55}$$

Thus the traces (3.51) correspond to the closed string sector that belongs to the each Chan-Paton factor. Thus the generation of the lower-dimensional D-branes' charges is induced by the above Wess-Zumino terms which are characteristic of the brane-antibrane systems.

Switching on the above operators, we obtain the following boundary states

$$|B(\{\alpha_i\}), +\rangle^{(0)}_{\mathrm{NSNS}} = 16 \sum_{\vec{w}_\phi \in \mathbb{Z}^8} \left[ K\left(\{m_i\} = \{w^i_\phi\}\right)\right] |\vec{0}, 2\vec{w}_\phi\rangle^{(0)}_{\mathrm{NSNS}}\,, \tag{3.56}$$

$$\begin{aligned} |B(\{\alpha_i\}), +\rangle^{(0)}_{\mathrm{RR}} &= 16 \sum_{\vec{w}_\phi \in \mathbb{Z}^8} \left[ K\left(\{m_i\} = \left\{w^i_\phi + \frac{1}{2}\right\}\right)\right] \times \\ &\qquad \times \left|\vec{0}, 2\vec{w}_\phi + \vec{1}, +\right\rangle^{(0)}_{\mathrm{RR}}\,, \end{aligned} \tag{3.57}$$

where

$$\begin{aligned} K(\{m_i\}) = {} & \prod_{i=1}^{8} \cos\left(\pi \alpha_i m_i\right) + \\ & + \left[ \prod_{i=1}^{6} \cos\left(\pi \alpha_i m_i\right) \cdot \sin\left(\pi \alpha_7 m_7\right) \sin\left(\pi \alpha_8 m_8\right) + 27 \text{ terms} \right] + \\ & + \left[ \prod_{i=1}^{4} \cos\left(\pi \alpha_i m_i\right) \cdot \prod_{j=5}^{8} \sin\left(\pi \alpha_j m_j\right) + 69 \text{ terms} \right] + \\ & + \left[ \cos\left(\pi \alpha_1 m_1\right) \cos\left(\pi \alpha_2 m_2\right) \cdot \prod_{i=3}^{8} \sin\left(\pi \alpha_i m_i\right) + 27 \text{ terms} \right] + \\ & + \prod_{i=1}^{8} \sin\left(\pi \alpha_i m_i\right). \end{aligned} \tag{3.58}$$

We have $|B\left(\{0\}\right), +\rangle_{\mathrm{NSNS,RR}} = |\mathrm{D8}\text{-}\overline{\mathrm{D8}}, +\rangle_{\mathrm{NSNS,RR}}$.

At $\alpha_i = 1$ for all the $i$, the zero mode parts of the above boundary states become

$$\text{NSNS sector} \quad \sum_{\vec{w}_\phi \in \mathbb{Z}^8} (-1)^{w_\phi^1 + \cdots + w_\phi^8} \, |\vec{0}, 2\,\vec{w}_\phi\rangle^{(0)}_{\text{NSNS}} \,, \tag{3.59}$$

$$\text{RR} \quad \text{sector} \quad \sum_{\vec{w}_\phi \in \mathbb{Z}^8} (-1)^{w_\phi^1 + \cdots + w_\phi^8} \left|\vec{0}, 2\,\vec{w}_\phi + \vec{1}, +\right\rangle^{(0)}_{\text{RR}} \,. \tag{3.60}$$

The boundary state of this system is rewritten as follows:

$$\left|\text{D0} - \overline{\text{D0}}, +\right\rangle^{(0)}_{\text{NSNS}} = 16 \sum_{\vec{n}_Y \in \mathbb{Z}^8} |\vec{n}_Y, \vec{0}\rangle^{(0)}_{\text{NSNS}} \,, \tag{3.61}$$

$$\left|\text{D0} - \overline{\text{D0}}, +\right\rangle^{(0)}_{\text{RR}} = 16 \sum_{\vec{n}_Y \in \mathbb{Z}^8} \left|\vec{n}_Y + \overrightarrow{1/2}, \vec{0}, +\right\rangle^{(0)}_{\text{RR}} \,. \tag{3.62}$$

These boundary states satisfy the definition equation (the boundary conditions) for the D0-$\overline{\text{D0}}$ system.

Again, the extra phase factor $(-1)^{w_\phi^1 + \cdots + w_\phi^8}$ changes the boundary condition from D8-$\overline{\text{D8}}$ to D0-$\overline{\text{D0}}$. This corresponds to the fact that D0-branes and $\overline{\text{D0}}$-branes are produced at each choices of

$$(x_{2k+1}, x_{2k+2}) = (0, 0) \qquad \text{or} \qquad (\pi, \pi) \,. \tag{3.63}$$

Around the above points, there exists a soliton (anti-soliton) if the number of coordinate pairs taking the value $(\pi, \pi)$ is even (odd). Finally evaluating the vacuum amplitudes for NSNS and RR sector with all the ghosts taking into account, it is easy to check the Cardy's constraint explicitly. Thus we have established in the closed string viewpoint that a tachyonic soliton on the D8-$\overline{\text{D8}}$ system produces a codimension eight D0-branes.

Next turn to the other points of $\alpha_i$. The tadpole cancellation restricts the admissible values to $\alpha_i = 0, 1$. We note the following basic observation.

$$\begin{array}{ccc}
 & \alpha = 0 & \alpha = 1 \\
\cos(\pi \alpha w) & 1 & (-1)^w \\
\sin(\pi \alpha w) & 0 & 0 \\
\cos\left(\pi \alpha \left(w + \dfrac{1}{2}\right)\right) & 1 & 0 \\
\sin\left(\pi \alpha \left(w + \dfrac{1}{2}\right)\right) & 0 & (-1)^w \,.
\end{array} \tag{3.64}$$

Then in the case when $2n$ of $\alpha_i$ is equal to 1, the system corresponds to the D$(8-2n)$-$\overline{\text{D}(8-2n)}$ system. The number of such configurations corresponds to the possible choice of the Chan-Paton factor in the closed string sector. For example, $28\,\Gamma_{ij}$ corresponds to the degree of freedom in order to set the direction of the codimension

2 among 8 directions. On the other hand, when the odd number of $\alpha_i$ is equal to 1, the boundary states in RR sector vanish and the system corresponds to a non-BPS D-branes.

Again we emphasize that the Chan-Paton factors in the closed string sector played the crucial role in our analysis.

# 4. Boundary state description of tachyon condensation in $\mathbb{T}^4/\mathbb{Z}_2$ orbifold theory

In this section we construct the boundary state description of tachyon condensations in the orbifold theory. We first discuss the decay mode from a D0-$\overline{\text{D0}}$ system to a non-BPS D1-brane in detail. Next we extend this result to the higher codimension cases. We also discuss the occurrence of the bose-fermi degeneracy [26, 27] in this formalism.

## 4.1 Construction of the boundary state

The boundary state which represents D0-$\overline{\text{D0}}$ in $\mathbb{T}^4/\mathbb{Z}_2$ orbifold at the radii $R_6 = \sqrt{2}$, $R_7 = R_8 = R_9 = R$ is given as follows:

$$|B\rangle = \frac{T_{p=0}}{2\sqrt{2}}(|U\rangle_{\text{NSNS}} + |U\rangle_{\text{RR}}) + \frac{N}{2\sqrt{2}}(|T\rangle_{\text{NSNS}} + |T\rangle_{\text{RR}})\,, \tag{4.1}$$

where $|U\rangle, |T\rangle$ denote the untwisted, twisted part of the boundary state. The normalization for twisted sector is determined by comparing the closed string vacuum amplitude with the open string one. The result is given by $N = 2^3\pi^{3/2}$.

The more detailed structure of each sector is written as

$$\begin{aligned}
|U\rangle_{\text{NSNS}} &= \frac{1}{2}\int\left(\frac{dk}{2\,\pi}\right)^5 \sum_{n_6,n_7,n_8,n_9} \frac{1+(-1)^{n_6}}{(2\,\pi\sqrt{2})(2\,\pi R)^3}\Big[\,\big|U,+,k^i,n\big\rangle_{\text{NSNS}} - \\
&\qquad\qquad - \big|U,-,k^i,n\big\rangle_{\text{NSNS}}\Big], \\
|U\rangle_{\text{RR}} &= 2\int\left(\frac{dk}{2\pi}\right)^5 \sum_{n_6,n_7,n_8,n_9} \frac{1-(-1)^{n_6}}{(2\pi\sqrt{2})(2\pi R)^3}\Big[\,\big|U,+,k^i,n\big\rangle_{\text{RR}} + \big|U,-,k^i,n\big\rangle_{\text{RR}}\Big], \\
|T\rangle_{\text{NSNS}} &= \int\left(\frac{dk}{2\pi}\right)^5 \Big[\,\big|T_1,+,k^i\big\rangle_{\text{NSNS}} + \big|T_1,-,k^i\big\rangle_{\text{NSNS}} + \big|T_2,+,k^i\big\rangle_{\text{NSNS}} + \\
&\qquad\qquad + \big|T_2,-,k^i\big\rangle_{\text{NSNS}}\Big], \qquad\qquad (4.2) \\
|T\rangle_{\text{RR}} &= \int\left(\frac{dk}{2\pi}\right)^5 \Big[\,\big|T_1,+,k^i\big\rangle_{\text{RR}} + \big|T_1,-,k^i\big\rangle_{\text{RR}} + \big|T_2,+,k^i\big\rangle_{\text{RR}} + \big|T_2,-,k^i\big\rangle_{\text{RR}}\Big],
\end{aligned}$$

where we set $1 \le i \le 5$ and $|T_1\rangle, |T_2\rangle$ represent the twisted sector boundary states corresponding to two different fixed points. As we will explain briefly in the appendix A, $|U,\gamma,k^i,n\rangle$ and $|T_1,+,k^i\rangle$, $|T_2,+,k^i\rangle$ are defined by the conditions (A.4) expanding the fields $(Y,\psi)$ in each Hilbert space.

Next we need to rewrite the above boundary state in terms of $(\phi, \eta)$ in order to describe the tachyon condensation as discussed in the previous section. For the untwisted sector the procedure is almost the same and the result are as follows (we show below only the relevant modes which correspond to $x^6$ direction and omit the superscript 6 in this subsection)

$$
\begin{aligned}
|U,+\rangle_{\rm NSNS} &= \exp\left(\sum_{n\in\mathbb{Z}} \frac{1}{n}\,\alpha_{-n}\,\tilde{\alpha}_{-n} - i \sum_{r\in\mathbb{Z}+\frac{1}{2}} \psi_{-r}\,\tilde{\psi}_{-r}\right) 2\sum_{n_Y} |2\,n_Y, 0\rangle^{(0)}_{\rm NSNS} \\
&= \exp\left(\sum_{n\in\mathbb{Z}} -\frac{1}{n}\,\phi_{-n}\,\tilde{\phi}_{-n} + i \sum_{r\in\mathbb{Z}+\frac{1}{2}} \eta_{-r}\,\tilde{\eta}_{-r}\right) 2\sum_{w_\phi} (-1)^{w_\phi}\,|0, w_\phi\rangle^{(0)}_{\rm NSNS}\,, \\
|U,+\rangle_{\rm RR} &= \exp\left(\sum_{n\in\mathbb{Z}} \frac{1}{n}\alpha_{-n}\tilde{\alpha}_{-n} - i \sum_{r\in\mathbb{Z}} \psi_{-r}\,\tilde{\psi}_{-r}\right) 2\sum_{n_Y} |2\,n_Y+1, 0, +\rangle^{(0)}_{\rm RR} \qquad (4.3)\\
&= \exp\left(\sum_{n\in\mathbb{Z}} -\frac{1}{n}\,\phi_{-n}\,\tilde{\phi}_{-n} + i \sum_{r\in\mathbb{Z}} \eta_{-r}\,\tilde{\eta}_{-r}\right) 2\sum_{w_\phi} (-1)^{w_\phi} \left|0, w_\phi + \frac{1}{2}, +\right\rangle^{(0)}_{\rm RR}.
\end{aligned}
$$

Next let us turn to the twisted sector. The twist operator $\sigma$ which map the untwisted sector into twisted sector is needed [32]. A candidate for such an operator is given as

$$
\sigma = e^{i\frac{1}{\sqrt{2}}(\phi_R - \tilde{\phi}_L)}\,, \qquad (4.4)
$$

which has the desired singular property as

$$
\psi(z)\,\sigma(0) \sim O(z^{-1/2})\,, \qquad \partial X(z)\,\sigma(0) \sim O(z^{-1/2})\,. \qquad (4.5)
$$

This operator leads to the correct boundary condition of twisted sector boundary state. Then we can rewrite the twisted sector boundary state as

$$
\begin{aligned}
|T,+\rangle_{\rm NSNS} &= \exp\left(\sum_{n\in\mathbb{Z}} \frac{1}{n+1/2}\,\alpha_{-(n+1/2)}\,\tilde{\alpha}_{-(n+1/2)} - i \sum_{r\in\mathbb{Z}} \psi_{-r}\,\tilde{\psi}_{-r}\right) \times \\
&\quad \times \left\{ |T_1\rangle^{(0)}_{\rm NSNS} + |T_2\rangle^{(0)}_{\rm NSNS} \right\} \\
&= \exp\left(\sum_{n\in\mathbb{Z}} -\frac{1}{n}\,\phi_{-n}\,\tilde{\phi}_{-n} + i \sum_{r\in\mathbb{Z}+\frac{1}{2}} \eta_{-r}\tilde{\eta}_{-r}\right) \sum_{w_\phi} (-1)^{w_\phi} \left| w_\phi + \frac{1}{2} \right\rangle^{(0)}_{\rm NSNS}\,, \\
|T,+\rangle_{\rm RR} &= \exp\left(\sum_{n\in\mathbb{Z}} \frac{1}{n+1/2}\,\alpha_{-(n+1/2)}\,\tilde{\alpha}_{-(n+1/2)} - i \sum_{r\in\mathbb{Z}+\frac{1}{2}} \psi_{-r}\,\tilde{\psi}_{-r}\right) \times \\
&\quad \times \left\{ |T_1\rangle^{(0)}_{\rm RR} + |T_2\rangle^{(0)}_{\rm RR} \right\} \\
&= \exp\left(\sum_{n\in\mathbb{Z}} -\frac{1}{n}\,\phi_{-n}\,\tilde{\phi}_{-n} + i \sum_{r\in\mathbb{Z}} \eta_{-r}\,\tilde{\eta}_{-r}\right) \sum_{w_\phi} (-1)^{w_\phi} |w_\phi\rangle^{(0)}_{\rm RR}\,. \qquad (4.6)
\end{aligned}
$$

Notice that this transformation or "bosonization" procedure can be verified by showing the bosonized boundary state does indeed satisfy the boundary condition of the original one as in section 3. It is also easy to see that the vacuum amplitude doesn't change by the bosonization using the relations (3.35).

### 4.2 Tachyon condensation

Since we have constructed the boundary state of D0-$\overline{\text{D0}}$ system in terms of $(\phi, \eta)$, it is straightforward to determine the boundary state which describe the tachyon condensation process in that system. The Wilson line corresponding to the tachyon condensation discussed in section 2 can be written as

$$W = \mathrm{Tr}\,\exp\left(\frac{i}{2\sqrt{2}}\,\alpha \oint \partial\phi \otimes \sigma_1\right) = \cos(\pi w_\phi\, \alpha)\,. \tag{4.7}$$

Then the effect of the tachyon condensation appears at the coefficients in front of the zero mode parts as in section 3. If we consider the point $\alpha = 1$, which corresponds to the maximal condensation, then the untwisted RR-sector and the twisted NSNS-sector vanish. The untwisted NSNS-sector and the twisted RR-sector become as follows:

$$\begin{aligned}
|U,+\rangle_{\mathrm{NSNS}} &= \exp\left(\sum_{n\in\mathbb{Z}} -\frac{1}{n}\,\phi_{-n}\,\tilde{\phi}_{-n} + i\sum_{r\in\mathbb{Z}+\frac{1}{2}} \eta_{-r}\,\tilde{\eta}_{-r}\right) 2\sum_{w_\phi} |w_\phi\rangle^{(0)}_{\mathrm{NSNS}} \\
&= \exp\left(\sum_{n\in\mathbb{Z}} -\frac{1}{n}\,\alpha_{-n}\,\tilde{\alpha}_{-n} + i\sum_{r\in\mathbb{Z}+\frac{1}{2}} \psi_{-r}\,\tilde{\psi}_{-r}\right) 2\sum_{w_Y} |w_Y\rangle^{(0)}_{\mathrm{NSNS}}\,, \\
|T,+\rangle_{\mathrm{RR}} &= \exp\left(\sum_{n\in\mathbb{Z}} -\frac{1}{n}\,\phi_{-n}\,\tilde{\phi}_{-n} + i\sum_{r\in\mathbb{Z}} \eta_{-r}\,\tilde{\eta}_{-r}\right) \sum_{w_\phi} |w_\phi\rangle^{(0)}_{\mathrm{RR}} \\
&= \exp\left(\sum_{n\in\mathbb{Z}} -\frac{1}{n+1/2}\,\alpha_{-(n+1/2)}\,\tilde{\alpha}_{-(n+1/2)} + i\sum_{r\in\mathbb{Z}+\frac{1}{2}} \psi_{-r}\,\tilde{\psi}_{-r}\right) \times \\
&\quad \times \left\{ |T_1\rangle^{(0)}_{\mathrm{RR}} + |T_2\rangle^{(0)}_{\mathrm{RR}} \right\},
\end{aligned} \tag{4.8}$$

where we have "rebosonized" the expression using the basis $(Y, \psi)$. Note that the boundary condition is changed into that of D1-brane because of the extra phase $\cos(\pi w_\phi) = (-1)^{w_\phi}$.

Now it is obvious[17] that the above boundary state is the same as that of a non-BPS D1-brane [8] stretching between the fixed points.

[17] If we start a D0-$\overline{\text{D0}}$ which has the different relative twisted charge, then we can show by using almost the same procedure that the final object is a non-BPS D1-brane with a $\mathbb{Z}_2$ Wilson line after the tachyon condensation.



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In this way the tachyon condensation process from D0-$\overline{\text{D0}}$ to a non-BPS D1-brane (and also its reverse if we replace $\alpha$ with $1-\alpha$) is explicitly shown by using boundary state formalism. It would be an interesting fact that the twisted sector of $(Y,\psi)$ can be expressed by using the untwisted sector of another field basis$(\phi,\eta)$ and this is crucial in the above discussion of the tachyon condensation in the orbifold theory. This fact will also become important if we consider tachyon condensation processes in other orbifold theories.

### 4.3 Generalization to the higher codimension case

Then we will be interested in the higher codimension cases. As we will show below, such generalizations are not so difficult in our boundary state formalism and the results remain almost the same as in section 3. Therefore the discussion is short.

To make things clear we consider the tachyon condensation that changes two D4-$\overline{\text{D4}}$ pairs into two D0-$\overline{\text{D0}}$ pairs (codimension four). Here D4-$\overline{\text{D4}}$ system has appropriate $\mathbb{Z}_2$ Wilson lines in the same sense of section 3. This process includes the decay modes into two D2-$\overline{\text{D2}}$ pairs. First let us define the coordinates of $\mathbb{T}^4$ as $(x^6,x^7,x^8,x^9)$ and their $\pi/4$ rotated coordinates as $(y^6,y^7,y^8,y^9)$. We also take the radii of $\mathbb{T}^4$ as $R^6=R^7=R^8=R^9=1$ in terms of the coordinates $(x^6,x^7,x^8,x^9)$ as in the previous discussion in flat space. It is important to note that this D4-$\overline{\text{D4}}$ system can be described in terms of $(y^6,y^7,y^8,y^9)$ as a D4-$\overline{\text{D4}}$ system on $\mathbb{T}^4/\mathbb{Z}_2$ of which radii are all $\sqrt{2}$ with the projection $(-1)^F h_6h_7h_8h_9=1$ as in the case of flat space. At this radius we can change the basis $(Y^i,\psi^i)$ into $(\phi^i,\eta^i)$ by the bosonization procedures, which are trivial generalizations of eqs. (4.3) and (4.6). Then we can describe the tachyon condensation processes and let us denote the corresponding four parameters as $\alpha_1,\alpha_2,\alpha_3,\alpha_4$. Notice that in order to get four parameters[18] corresponding to the marginal deformation in the four directions we should start with not one but two pieces of D4-$\overline{\text{D4}}$. At the point $\alpha_1=\alpha_2=\alpha_3=\alpha_4=1$ both the untwisted and the twisted boundary states gain the same extra phase factor $(-1)^{w_6+w_7+w_8+w_9}$ and the boundary conditions along $(y^6,y^7,y^8,y^9)$ are reversed. Then we get two D0-branes and two anti D0-branes which sit at $(x^6,x^7,x^8,x^9)=(0,0,0,0),(\pi,\pi,\pi,\pi)$ and $(\pi,\pi,0,0),(0,0,\pi,\pi)$, respectively. In this way we find that the tachyon condensation of brane-antibrane system in $\mathbb{T}^4/\mathbb{Z}_2$ orbifold can be treated almost in the same way as in flat space except the treatment of the twisted sector.

### 4.4 Comments on bose-fermi degeneracy

Finally let us discuss the relation between the boundary state description in this section and the bose-fermi degeneracy [26]. First we compute the vacuum amplitude of the system (4.3) and (4.6) with the insertion of the Wilson line (4.7) using $(\phi,\eta)$

[18] If we started with one D4-$\overline{\text{D4}}$, then we would only get the decay modes into a D2-$\overline{\text{D2}}$ and the codimension four configuration is impossible.

field representation for $x^6$ direction. The result is

$$\langle B(\alpha)|\Delta|B(\alpha)\rangle = \frac{V}{16}\int ds\left(\frac{1}{2\pi s}\right)^{5/2} \times$$
$$\times\left[\frac{\pi^3\, 2^{5/2}}{R^3}\left\{\sum_{w_\phi,\vec{n}_Y} q^{{w_\phi}^2+\frac{n_Y^2}{2R^2}}\cos^2(\pi\alpha\, w_\phi)\frac{f_3(q)^8}{f_1(q)^8} - \right.\right.$$
$$-\sum_{\vec{n}_Y} q^{\frac{n_Y^2}{2R^2}}\frac{\sqrt{2}\, f_3(q)f_4(q)^7}{f_2(q)f_1(q)^7} -$$
$$\left.-\sum_{w_\phi,\vec{n}_Y}\cos^2\left(\pi\alpha\left(w_\phi+\frac{1}{2}\right)\right) q^{(w_\phi+1/2)^2+\frac{n_Y^2}{2R^2}}\frac{f_2(q)^8}{f_1(q)^8}\right\} +$$
$$+\, 2^6\pi^3\left\{\sum_{w_\phi}\cos^2\left(\pi\left(w_\phi+\frac{1}{2}\right)\alpha\right) q^{(w_\phi+1/2)^2}\frac{f_3(q)^5 f_2(q)^3}{\sqrt{2}f_1(q)^5 f_4(q)^3} - \right.$$
$$\left.\left.-\sum_{w_\phi}\cos^2(\pi w_\phi\alpha) q^{w_\phi^2}\frac{f_2(q)^5 f_3(q)^3}{\sqrt{2}f_1(q)^5 f_4(q)^3}\right\}\right], \qquad (4.9)$$

where $\vec{n}_Y = (n_Y^7, n_Y^8, n_Y^9) \in \mathbb{Z}^3$ are momenta in the directions of $(y^7, y^8, y^9)$. We can see that this amplitude does vanish if $R = 1/\sqrt{2}, \alpha = 1$ and this phenomenon of non-BPS D1-brane is called bose-fermi degeneracy [26]. Below we would like to discuss this from the viewpoint of the boundary state.

The particular radii of torus $R = 1/\sqrt{2}$ enable us to perform further bosonization procedures in the direction of $x^7, x^8, x^9$. The result is as follows (we only show the zero modes and oscillators which correspond to $x^6, x^7, x^8, x^9$)

$$|U,+\rangle_{\rm NSNS} = \exp\left[\sum_{n\in\mathbb{Z}}\frac{1}{n}\left(-\alpha^6_{-n}\,\tilde{\alpha}^6_{-n} + \sum_{i=7,8,9}\alpha^i_{-n}\,\tilde{\alpha}^i_{-n}\right) - \right.$$
$$\left. - i\sum_{r\in\mathbb{Z}+\frac{1}{2}}\left(-\psi^6_{-r}\,\tilde{\psi}^6_{-r} + \sum_{i=7,8,9}\psi^i_{-r}\,\tilde{\psi}^i_{-r}\right)\right] \times$$
$$\times\, 2\sum_{w_Y^6,\vec{n}_Y}\left|w_Y^6, n_Y^7, n_Y^8, n_Y^9\right\rangle^{(0)}_{\rm NSNS}$$
$$= \exp\left[\sum_{n\in\mathbb{Z}}\frac{1}{n}\left(-\phi^6_{-n}\,\tilde{\phi}^6_{-n} - \sum_{i=7,8,9}\phi^i_{-n}\,\tilde{\phi}^i_{-n}\right) - \right.$$
$$\left. - i\sum_{r\in\mathbb{Z}+\frac{1}{2}}\left(-\eta^6_{-r}\,\tilde{\eta}^6_{-r} - \sum_{i=7,8,9}\eta^i_{-r}\,\tilde{\eta}^i_{-r}\right)\right] \times$$
$$\times\, 2\sum_{w_\phi}(-1)^{w_\phi^7+w_\phi^8+w_\phi^9}\left|w_\phi^6, 2\,w_\phi^7, 2\,w_\phi^8, 2\,w_\phi^9\right\rangle^{(0)}_{\rm NSNS},$$

$$
\begin{aligned}
|T,+\rangle_{\mathrm{RR}} &= \exp\left[\sum_{n\in\mathbb{Z}} \frac{1}{n+1/2}\left(-\alpha^6_{-n-1/2}\,\tilde{\alpha}^6_{-n-1/2} + \sum_{i=7,8,9} \alpha^i_{-n-1/2}\,\tilde{\alpha}^i_{-n-1/2}\right) - \right.\\
&\qquad \left. - i \sum_{r\in\mathbb{Z}+\frac{1}{2}} \left(-\psi^6_{-r}\,\tilde{\psi}^6_{-r} + \sum_{i=7,8,9} \psi^i_{-r}\,\tilde{\psi}^i_{-r}\right)\right] \left\{|T_1\rangle^{(0)}_{\mathrm{RR}} + |T_2\rangle^{(0)}_{\mathrm{RR}}\right\} ,\\
&= \exp\left[\sum_{n\in\mathbb{Z}} \frac{1}{n}\left(-\phi^6_{-n}\,\tilde{\phi}^6_{-n} - \sum_{i=7,8,9} \phi^i_{-n}\,\tilde{\phi}^i_{-n}\right) - \right.\\
&\qquad \left. - i \sum_{r\in\mathbb{Z}} \left(-\eta^6_{-r}\,\tilde{\eta}^6_{-r} - \sum_{i=7,8,9} \eta^i_{-r}\,\tilde{\eta}^i_{-r}\right)\right] \times\\
&\qquad \times 2 \sum_{w_\phi} (-1)^{w^7_\phi + w^8_\phi + w^9_\phi} \left|w^6_\phi, 2\,w^7_\phi, 2\,w^8_\phi, 2\,w^9_\phi, +\right\rangle^{(0)}_{\mathrm{RR}} .
\end{aligned}
\tag{4.10}
$$

This expression shows that the twisted RR-sector in terms of $(Y,\psi)$ is rewritten to have the same form as the untwisted RR-sector of D4-brane in terms of $(\phi,\eta)$.[19] If we use the basis $(Y,\psi)$ for NSNS-sector and $(\phi,\eta)$ for RR-sector, each open string vacuum amplitude of NS-sector and R-sector cancels each other and the occurrence of the bose-fermi degeneracy is explicitly shown. Therefore we can say that the bosonization procedures at critical radius are crucial in the bose-fermi degeneracy.

## 5. Conclusions

In this paper we have shown explicitly in the boundary state formalism that the tachyon condensation in $2^{k-1}$ pieces of D$(p+2k)$-$\overline{\mathrm{D}(p+2k)}$ at critical radii produces $2^{k-1}$ pieces of D$p$-$\overline{\mathrm{D}p}$. Locally this means that a codimension $2\,k$ soliton of the tachyon field configuration corresponds to a D$p$-brane (or $\overline{\mathrm{D}p}$). We have also verified this results in $\mathbb{T}^4/\mathbb{Z}_2$ orbifold theory. Note that in these cases there are no gauge fields on the world volume. But the generations of lower D-brane charges indeed occur due to the Wess-Zumino terms which are peculiar to brane-antibrane systems [30]. In the boundary state description we have succeeded to see these phenomena explicitly.

In the process of the explicit calculations we have found two remarkable facts. The first is that the consistency with the open string picture (or Cardy's condition) requires the closed string sectors should have non-trivial Chan-Paton factors. This somewhat strange phenomenon only occurs if we discuss interactions of closed strings with brane-antibrane systems or non-BPS D-branes. These Chan-Paton factors also ensure the Wess-Zumino coupling proposed in [30].

[19] This expression also implies that the original non-BPS D1-brane in $\mathbb{T}^4/\mathbb{Z}_2$ can be thought as a BPS D4-brane in terms of the field $(\phi,\eta)$ with a "wrong GSO projection" $(-1)^F = {I_4}^{\phi}(-1)^{F^\phi} = 1$, though the essence of this interpretation is not clear.

The second one is the fact in the case of $\mathbb{T}^4/\mathbb{Z}_2$ orbifold we can treat the twisted sector boundary state in the same way as the untwisted one by changing the field basis (or by “bosonization” procedure). This enables us to construct the boundary state which describe the tachyon condensation in the orbifold theory. Another application of this fact is the investigation of bose-fermi degeneracy [26, 27]. At the point where the degeneracy occurs the boundary state of a non-BPS D1-brane becomes very much like that of a BPS D-brane by using the bosonization procedure. Naively it seems that a sort of a symmetry is enhanced at this particular moduli, but it is difficult to see this explicitly even in our formalism. We leave this as a future problem. So far the tachyon condensation in four-dimensional orbifold theories other than $\mathbb{T}^4/\mathbb{Z}_2$ have not been discussed. If one try to construct the marginal deformations of BCFT in them, something like the previous bosonization procedures of the boundary state will be required.

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## A. Structure of boundary state

Here we give our CFT conventions and a short review of boundary states for general D$p$-branes with or without the orbifold projection. Remember that we have used the light cone formulation [29, 13] and ignored[20] the non-zero modes of the fields $(Y^8, Y^9, \psi^8, \psi^9)$ in the case of D2-$\overline{\text{D2}}$ system. We use almost the same conventions as Sen's except the detailed normalizations.

### A.1 CFT conventions

We define $z = e^{-i\sigma_1+\sigma_2}$ as the cylindrical coordinate of the world sheet and $w = \sigma_1 + i\,\sigma_2$ as its radial plane coordinate. First we list the mode expansions of $(Y, \psi)$ fields:

$$
\begin{aligned}
Y^i_R(z) &= y^i_R - \frac{i}{2}\, p^i_{YR} \ln z + \frac{i}{\sqrt{2}} \sum_{n\neq 0} \frac{1}{n} \frac{(\alpha_Y)^i_n}{z^n}\,, \\
Y^i_L(\bar{z}) &= y^i_L - \frac{i}{2}\, p^i_{YL} \ln \bar{z} + \frac{i}{\sqrt{2}} \sum_{n\neq 0} \frac{1}{n} \frac{(\tilde{\alpha}_Y)^i_n}{\bar{z}^n}\,, \\
\psi^i_R(z) &= i^{1/2} \sum_{r\in\mathbb{Z}+\nu} \frac{\psi^i_r}{z^{r+1/2}}\,, \qquad \psi^i_L(\bar{z}) = i^{-1/2} \sum_{r\in\mathbb{Z}+\nu} \frac{\tilde{\psi}^i_r}{\bar{z}^{r+1/2}}\,, \\
&\Big(\hat{p}^i_{YR} = \sqrt{2}\,(\alpha_Y)^i_0\,, \qquad \hat{p}^i_{YL} = \sqrt{2}\,(\tilde{\alpha}_Y)^i_0\Big)\,,
\end{aligned}
\tag{A.1}
$$

[20] In the case of the orbifold theory discussed in section 4, we ignored the non-zero modes of $(X^4, X^5, \psi^5, \psi^6)$.

where $\nu = 1/2$ represents NS-sector and $\nu = 0$ R-sector and we set $i = 0 \sim 9$.

Then the OPE relations (2.6) are equivalent to the following (anti)commutation relations for modes:

$$\begin{aligned}
\left[y_L^i, p_{YL}^j\right] &= \left[y_R^i, p_{YR}^j\right] = i\,\eta^{ij}\,, \\
\left[(\alpha_Y)_m^i, (\alpha_Y)_n^j\right] &= \left[(\tilde{\alpha}_Y)_m^i, (\tilde{\alpha}_Y)_n^j\right] = m\,\delta_{m,-n}\,\eta^{ij}\,, \\
\left\{\psi_r^i, \psi_s^j\right\} &= \left\{\tilde{\psi}_r^i, \tilde{\psi}_s^j\right\} = \delta_{r,-s}\,\eta^{ij}\,,
\end{aligned} \tag{A.2}$$

where $\eta^{ij} = \delta^{ij}$ for $i = 1 \sim 9$ and $\eta^{ij} = -\delta^{ij}$ for $i = 0$. The vacuum of these modes is defined as $|\Omega\,\rangle^{(0)}_{\mathrm{NSNS}}$. If we compactify the coordinates $y^i$ on torus (radii $R^i$) then the momenta are quantized as follows:

$$p_{YR}^i = \frac{n_Y^i}{R^i} + R^i w_Y^i\,, \qquad p_{YL}^i = \frac{n_Y^i}{R^i} - R^i w_Y^i\,, \tag{A.3}$$

where $n_Y^i$ and $w_Y^i$ denote K.K. modes and winding modes.

We have also used the fields $(X, \chi)$ and $(\phi, \eta)$ as another bases. The mode expansions and commutation relations of these fields are defined in the same way.

### A.2 Definition of boundary states

A boundary state of D$p$-brane is defined by the following boundary conditions[21] in the closed string Hilbert space:

$$\begin{aligned}
\partial_2\, Y^\mu(w)|_{\sigma_2=0}\,|\mathrm{D}p,\gamma\,\rangle &= 0\,, \quad (\mu = 0 \sim p)\,, \\
\partial_1 Y^i(w)|_{\sigma_2=0}\,|\mathrm{D}p,\gamma\,\rangle &= 0\,, \quad (i = p+1 \sim 7)\,, \\
(\psi_R^\mu(w) - i\,\gamma\,\psi_L^\mu(\bar{w}))\,|_{\sigma_2=0}\,|\mathrm{D}p,\gamma\,\rangle &= 0\,, \quad (\mu = 0 \sim p)\,, \\
\left(\psi_R^i(w) + i\,\gamma\,\psi_L^i(\bar{w})\right)|_{\sigma_2=0}\,|\mathrm{D}p,\gamma\,\rangle &= 0\,, \quad (i = p+1 \sim 7)\,,
\end{aligned} \tag{A.4}$$

where $\gamma = +, -$ is the spin structure on the boundary and the GSO projection of the closed string determines the correct linear combination of these spin structures. If we expand the left-hand side of eq. (A.4), we get

$$\begin{aligned}
((\alpha_Y)_n^\mu + (\tilde{\alpha}_Y)_{-n}^\mu)\,|\mathrm{D}p,\gamma\,\rangle &= 0\,, \quad (\mu = 0 \sim p)\,, \\
\left((\alpha_Y)_n^i - (\tilde{\alpha}_Y)_{-n}^i\right)|\mathrm{D}p,\gamma\,\rangle &= 0\,, \quad (i = p+1 \sim 7)\,, \\
\left(\psi_r^\mu - i\,\gamma\,\tilde{\psi}_{-r}^\mu\right)|\mathrm{D}p,\gamma\,\rangle &= 0\,, \quad (\mu = 0 \sim p)\,, \\
\left(\psi_r^i + i\,\gamma\,\tilde{\psi}_{-r}^i\right)|\mathrm{D}p,\gamma\,\rangle &= 0\,, \quad (i = p+1 \sim 7)\,.
\end{aligned} \tag{A.5}$$

These conditions are easy to solve by using the commutation relations (A.1) and (A.2). Notice that for a BPS D-brane the boundary state consists of the NSNS-sector and RR-sector and the correct linear combination of them should be deter-

[21] Of course the conditions remain the same if we replace $(Y, \psi)$ with $(X, \chi)$, because this procedure does not mix the Neumann and Dirichlet conditions.

mined by comparing its cylinder amplitude with that of open string (see [22]). For example in the case of a (BPS) D2-brane the boundary state is given as eq. (3.1). Also note that for a non-BPS D-brane there is no RR-sector.

Finally let us see the orbifold case briefly. In general the orbifold theories have twisted sectors in the closed string Hilbert space and therefore it is necessary to add twisted sector boundary states $|T\rangle$ to the untwisted one. The twisted sector boundary states are defined by the same equation (A.4), but the mode expansion is different from (A.1) because of the twisted boundary condition. In the case of $\mathbb{T}^4/\mathbb{Z}_2$ orbifold discussed in section 4, the mode expansion of $(Y^i, \psi^i)(i = 6, 7, 8, 9)$ is shifted by half integer. For example, the twisted sector boundary state of D0-$\overline{\text{D0}}$ is given as eq. (4.6), where we showed only the modes of $(Y^6, \psi^6)$. The correct linear combination of the twisted sector boundary states and the untwisted one is also determined by the calculations of the cylinder amplitude and this is called the Cardy's condition [23].



## B. Equivalence of a boundary state and its bosonized version

In sections 3 and 4 we have used bosonized (and fermionized) descriptions of boundary states at special radii. In [24] the authors calculate several one point functions in the codimension one case and show that the results are the same as those before the bosonization. As a further evidence of the equivalence here we prove that the bosonized boundary states discussed in section 3.1 satisfy the correct boundary conditions in the case of the tachyon condensation in D2-$\overline{\text{D2}}$ system. The other cases appeared in this paper can be treated almost in the same way.

### B.1 Cocycle factors

In order to prove the correct boundary conditions the detailed cocyle factors should be given explicitly. For example, the fermionization relations (2.5) are written incorporating the cocyle factors as

$$\begin{aligned}
\tau_i \otimes C_Y^i\left(\pm\sqrt{2}, 0\right) e^{\pm i\sqrt{2}Y_R^i}(z) &\cong \frac{1}{\sqrt{2}}\left(\xi_R^i \pm i\eta_R^i\right)(z)\,, \\
\tau_i \otimes C_Y^i\left(0, \pm\sqrt{2}\right) e^{\pm i\sqrt{2}Y_L^i}(\bar{z}) &\cong \frac{1}{\sqrt{2}}\left(\xi_L^i \pm i\eta_L^i\right)(\bar{z})\,,
\end{aligned} \tag{B.1}$$

where $\tau_i, C_Y^i(k_R, k_L)$ are both called cocycle factors. $\tau_i (i = 1, 2, 3)$ are $2 \times 2$ Pauli matrices. $C_Y^i(k_R, k_L)$ are defined by (for example see [33])

$$C_Y^i(k_R, k_L) \equiv \exp\left[\frac{1}{4}\pi\, i\left(k_{YR}^i - k_{YL}^i\right)\left(\hat{p}_{YR}^i + \hat{p}_{YL}^i\right)\right]. \tag{B.2}$$

In bosonization procedure, they are needed to guarantee correct (anti)commutation relations between various fields.

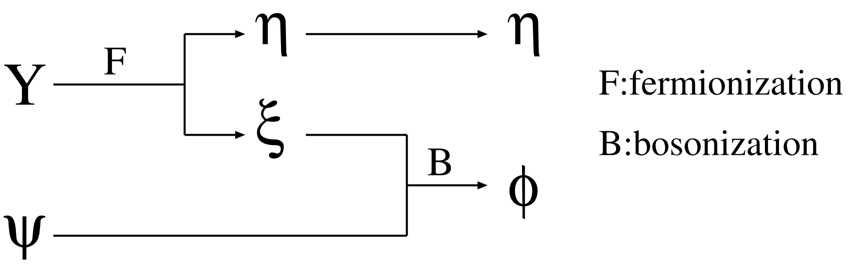


**Figure 3:** Flow of the bosonization.

Next step is the rebosonization of two fermions,

$$\begin{aligned}\frac{1}{\sqrt{2}}\left(\xi^i_R \pm i\,\psi^i_R\right)(z) &\cong \tilde{\tau}_i \otimes C^i_\phi\left(\pm\sqrt{2},0\right)\, e^{\pm i\sqrt{2}\,\phi^i_R}(z)\,,\\ \frac{1}{\sqrt{2}}\left(\xi^i_L \pm i\psi^i_L\right)(\bar{z}) &\cong \tilde{\tau}_i \otimes C^i_\phi\left(0,\pm\sqrt{2}\right)\, e^{\pm i\sqrt{2}\,\phi^i_L}(\bar{z})\,.\end{aligned} \tag{B.3}$$

In this way we accomplished changing variables from $(Y^i, \psi^i)$ to $(\phi^i, \eta^i)$(see figure 3).

### B.2 Proof of the correct boundary conditions

Now let us prove the facts that the bosonized boundary states satisfy the correct boundary conditions of the original ones and give a evidence that they are equivalent. We take the example of D2-$\overline{\text{D2}}$ system discussed in section 3. Then two types of equivalence should be proved. The first is that eqs. (3.12) and (3.13) are equivalent to eqs. (3.10) and (3.11), respectively. We can verify this by showing that eqs. (3.12) and (3.13) satisfy the boundary conditions eqs. (3.14) and (3.15). The second case is that $|B(\alpha=1,\beta=1),+\rangle$ (see eqs. (3.23) and (3.24)) is equivalent to the boundary state of D0-$\overline{\text{D0}}$ system. We can also prove this in the same way by showing eqs. (3.28) and (3.29). Since these four equations can be proven in the same way, we show the proof of (3.14) below.

First let us note that eqs. (3.12) and (3.13) satisfy

$$\left(\eta^i_R(w) - i\,\eta^i_L(\bar{w})\right)|_{\sigma_2=0}\left|\text{D2}-\overline{\text{D2}},+\right\rangle_{\text{NSNS,RR}} = 0\,, \tag{B.4}$$

and that we can replace $\partial_2 Y^i(w)|_{\sigma_2=0}$ with $(\phi,\eta)$ variables. Then eq. (3.14) can be rewritten as

$$\begin{aligned}&\partial_2 Y^i(w)|_{\sigma_2=0}\left|\text{D2}-\overline{\text{D2}},+\right\rangle_{\text{NSNS,RR}} =\\ &\quad= \tilde{\tau}_i \otimes \frac{\sqrt{z}}{2i}\,\eta^i_R(z)\left[\sqrt{z}\left\{C^i_\phi(\sqrt{2},0)e^{i\sqrt{2}\phi^i_R}(z) + C^i_\phi\left(-\sqrt{2},0\right)e^{-i\sqrt{2}\,\phi^i_R}(z)\right\}+\right.\\ &\qquad\qquad \left.+\ \sqrt{\bar{z}}\left\{C^i_\phi(0,\sqrt{2})e^{i\sqrt{2}\phi^i_L}(\bar{z}) + C^i_\phi(0,-\sqrt{2})e^{-i\sqrt{2}\phi^i_L}(\bar{z})\right\}\right]\times\\ &\qquad\times\left|\text{D2}-\overline{\text{D2}},+\right\rangle_{\text{NSNS,RR}}\,.\end{aligned} \tag{B.5}$$

The detail of the exponential is given as

$$: e^{i\sqrt{2}\,\phi^i_R}(z) : |_{\sigma_2=0} = \exp\left[i\,\sqrt{2}\,\phi_R\right] \exp\left[-\frac{i}{\sqrt{2}}\,p_R\,\sigma_1\right] \times \\ \times \exp\left[\sum_{n=1}^{\infty} \frac{1}{n}\,\phi_{-n}\,e^{-in\sigma_1}\right] \exp\left[-\sum_{n=1}^{\infty} \frac{1}{n}\,\phi_n\,e^{in\sigma_1}\right]. \quad (B.6)$$

If we note that eqs. (3.12) and (3.13) satisfy

$$\left(\phi_n + \tilde{\phi}_{-n}\right) \left|\mathrm{D2} - \overline{\mathrm{D2}}, +\right\rangle_{\mathrm{NSNS,RR}} = 0\,, \quad (B.7)$$

then it is easy to see that the first and the third, the second and the fourth term in eq. (B.5) cancel, respectively. The proof is almost the same as in the case of $|B(\alpha = 1, \beta = 1), +\rangle$ except that the $\mathbb{Z}_2$-phases $(-1)^{w^1_\phi + w^2_\phi}$ of eq. (3.27) play an important role for changing boundary conditions of $Y^1, Y^2$.

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